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On Domination in Matroids of Graphs

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Abstract: ***The concept of domination has increasingly found powerful extensions within the structural framework of matroid theory. This paper investigates the structural, algebraic, and algorithmic properties of domination in matroids, unifying classical network domination with matroidal invariants such as closures, independent sets, and fundamental circuits. We formalise the notion of a dominating configuration in a general matroid M = (E, I) and analyse the combinatorial parameters governing its minimal representations. Specifically, we characterize the complexity of computing the minimum-weight domination number across prominent matroid classes, including graphic matroids. By leveraging the matroid rank function r(M), we establish novel lower and upper bounds for the domination invariant.***

Keywords: Matroid Theory, Combinatorial Optimisation, Matroid Invariants, Graphic Matroids.
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I. INTRODUCTION

Matroid M is an ordered pair (V, M) where V is a nonempty set, not necessarily finite, whose elements are called vertices of M, and M is a collection of subsets called independent sets of *M, satisfying the following* independence axioms (IA) [4].

IA-1 $\phi \in$ M

IA-2 $X \in$ M and $Y \in X \Rightarrow Y \in$ M

IA-3 $X \in$ M, $Y \in$ M and $|X| > |Y| \Rightarrow \exists x \in X-Y$: $Y \cup \{x\} \in$ M

The set V is often called the ground set of *M.* Further, a maximal independent set of *M* and a subset of V not belonging to M is said to be dependent [6]. Hence, a minimal dependent subset of V is called a circuit of *M* [5]. The set of all bases of *M* is denoted by *CM denotes BM, and the set of all circuits of M.* A singleton subset $x \in$ M is called a loop in *M*. For terminology in matroid theory [2] and that for graphs and hypergraphs [1].

This paper introduces the notion of domination in matroids.

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Our motivation comes from an ever-growing literature on domination in graphs and hypergraphs, and from the fact that matroids can essentially be treated as certain hypergraphs [3].

II. DOMINATION IN MATROIDS

Definition 2.1. In any matroid M = (V, M), D ⊆ V is a dominating set if for every v ∈ V − D there exists a nonempty subset A ⊆ D such that A ∪ {v} ∈ C_M.

Given a matroid M = (V, M), A ⊆ V is said to depend on a if either x ∈ A contains a circuit of M that contains x. The closure of A, denoted by cl(A), is defined as the set of vertices that are dependent on A. Clearly, A ⊆ cl(A). A flat in M is then defined as a set A ⊆ V for which cl(A) = A. It may now be easily seen that the notion of a domset in M can be regarded as the other extreme of the notion of a flat in M since A is a domset of M ⇔ cl(A) − A = V − A ⇔ cl(A) = V. Hence, we denote by D (M) the set of all domsets of M.

Definition 2.2. Let M be any matroid and let v ∈ V. Then, the set C_M (v) = {C ∈ C_M: v ∈ C} is called the circuit neighbourhood of v, and its cardinality |CM (v)| is called the degree of v in M, denoted by d_M (v). A vertex u is called an isolate in M if d(u) = 0. Further, M is called locally finite if d(v) < ∞ for every v ∈ V (M).

Lemma 2.3. In any matroid M = (V, M) without isolates, for any minimal domset D, its complement V − D is a domset of M.

Proof: Under the hypothesis of the lemma, assume that V − D is not a domset of M. Then, by definition of a domset, there exists d ∈ D such that for no nonempty subset S ⊆ V − D one has S ∪ {d} ∈ C_M. However, since d is not an isolated in M it must be contained in a circuit A of M. Further, A must contain at least one vertex of D other than d, for otherwise, A−{d}: = A− dwould be a subset of V − D such that {d}∪ (A− d) ∈ C_M contrary to our deduction above that for no nonempty subset of V − D this is true. But then, D − d ∈ D(M), contradicting the minimality of D.

Lemma 2.4. In any isolate-free matroid M = (V, M), every dependent set has at least two vertices.

Proof: Suppose M has a dependent set S that has fewer than two vertices. Since S is dependent, S must be non-empty by virtue of IA − I. Therefore, S = {x} for some x ∈ M; otherwise, for any domset D of M there would be a nonempty A ⊆ D such that {x}∪ A ∈ C_M, which is preposterous. Next, let D be any minimal domset of M.

Therefore, for x ∈ D we must have a nonempty subset B ⊆ V

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$-$ D such that $\{x\} \cup B \in C_M$, which is again preposterous as $\{x\} \in C_M$. Thus, the result follows by contraposition.

Proposition 2.5. Let $M = (V, M)$ be any matroid such that every dependent subset of V contains a minimal one. Then, (i) every base of M is a minimal domset of M, and (ii) every independent domset of M is a base of M.

Proof (i): Let $D \in B_M$. Then, by definition, no superset of D is independent; hence, in particular, $D \cup \{v\}$is a dependent set for every $v \in V- D$. By hypothesis, $D_v := D \cup \{v\}$ contains a circuit A and then, according to Lemma 2.3, $|A| \geq 2$. Since D is independent, A is not a subset of D. Therefore, $v \in A$. Since $A \subseteq D_v$ and $|A| \geq 2$, $A\text{-}v \neq \emptyset$ and $A - v \subseteq D$; hence $\{v\} \cup (A - v) = A \in C_M$. Thus, $D \in D(M)$.

Further, we claim that D must in fact be a minimal domset of M. Suppose it is not. Then, there exists a proper domset B in D. Hence, by definition, it follows that for every $v \in V - B$ there exists a nonempty $A \subseteq B$ such that $A \cup \{v\} \in C_M$. Then, since $D - B \subseteq V - B$, in particular, for every $u \in D - B$ there must exist a nonempty $K \subseteq B$ such that $K \cup \{u\} \in CM$. But since $K \cup \{u\} \subseteq D$, we arrive at a contradiction, as $D \in M$. Thus, D must be a minimal domset of M.

(ii) Assume that D is an independent domset of M. Suppose that D is not a base of M. Then, there must exist an independent set D' that properly contains D so that there exists $v \in D' - D$. Since $D' - D \subseteq V - D$ and D is a domset of M, there must exist a nonempty subset B of D such that $B \cup \{v\} \in C_M$. But then, $B \cup \{v\}$ cannot be a subset of D' according to IA $-$ 2 since $D' \in M$. Hence, the proof follows by contraposition.

Proposition 2.6. In any locally finite matroid, every dependent set contains a minimal one.

Proof: Suppose the statement in the proposition is false. That means, there exists a locally finite matroid $M = (V, M)$ having a dependent set S_0 that contains no minimal dependent subset; thus, in particular, S_0 is not minimal. This implies S_0 contains a proper subset S_1 which is dependent. Since S_0 contains no minimal dependent subset, S_1 itself cannot be minimal. So, there must exist a proper dependent subset S_2 of S_1. Again, since $S_2 \subset S_1 \subset S_0$ and S_0 contains no minimal dependent subset, we see that S_2 itself cannot be minimal and hence must contain a proper dependent subset, S_3. Continuing in this manner, we obtain an infinite descending chain $S_0 \supset S_1 \supset S_2 \supset S_3 \supset \ldots$ of dependent subsets S_i of S_0, none of which is a circuit of M. Also, each of the so obtained S_i's must be nonempty, due to IA-1 and hence their descending subset property (DSP) implies that $S = \cap_{i\geq 0} S_i \neq \phi$. Further, S cannot be dependent, for, otherwise, S itself would be a circuit contained in S_0, contradicting our assumption about S_0. This implies $S \in M$. However, S cannot be a base of M for, otherwise, S being then a domset of M would imply that for every $u \in S_0 - S$ there exists a nonempty subset $C \subseteq S$ such that $C \cup \{u\} \in CM$. But, since $C \cup \{u\} \subseteq S_0$, the preceding conclusion violates our assumption about S_0. Therefore, there exists a base B that properly contains S. By IA-2, B contains none of the S_i's. Also, B is not a subset of any S_i, $i \geq 0$, for if $B \subseteq S_i$ for some i, then since B is a domset of M, for each $v \in S_i - B$ there exists a nonempty subset C of B such that $C \cup \{v\} \in C_M$ where $C \cup \{v\} \subseteq S_i \subset S_0$, a contradiction. Thus, $B \cap (V - S_0) \neq \phi$. So, we may choose vertices $u_i \in Si - 1 - (B \cup S_i)$ for each integer $i \geq 1$. Then, since B is a domset of M, for each $i \geq 1$, there must exist nonempty subsets $A_i \subset B$ such that $C_i = A_i \cup \{u_i\} \in CM$. Let $A = \cup i \geq 1 A_i$. Clearly, $A \subseteq B$.

If $A = B$, then $S = \cap_{i\geq 1} S_i \subset B = A = \cup_{i\geq 1} A_i$, whence it follows that every vertex x of S lies in Ci for infinitely many integers $i \geq 1$, implying that $d(x) = \infty$. Since $S \neq \phi$, this conclusion contradicts the hypothesis that M is locally finite. Thus, we conclude $A \subset B$. Then, by IA-2, we get $A \in M$. Also, since $|B| > |A|$, it follows by IA-3 that there exists $x \in B - A$ such that $A \cup \{x\} \in M$. Then, $\{C_i = A \cup \{x, u_i\} : i \geq 1\}$ is an infinite family of distinct dependent sets in M such that $x \in C_i$ for each $i \geq 1$. Now, since $C_i - u_i = A \cup \{x\} \in M$, it follows that $C_i \in C_M$ for every $i \geq 1$. This implies $d(x) = \infty$, which contradicts the hypothesis.

III. SOME NEW DIRECTIONS IN MATROID THEORY

We see that the ground set of any locally finite matroid can be partitioned into at least two domsets, but we cannot in general use" minimal domsets" in place of" domsets" in this statement.

Partitioning the ground set of a matroid into bases is a well-considered problem in matroid theory. In fact, it is well known that not all matroids are partitionable in this manner. Hence, a natural problem is to find some $k \geq 2$ pairwise disjoint bases in a given matroid.

Theorem 3.1. A finite matroid $M = (V, M)$ with rank function ρ has k pairwise non-intersecting bases if and only if $k\rho(A) + |V - A| \geq k\rho(V), \forall A \subseteq V$.

Proof: Since we now know that bases in a matroid are its independent domsets, which, in view of Proposition 2.7(i), are minimal domsets in the matroid, one may pose the general problem to find the maximum number of pairwise disjoint domsets in a finite matroid. We call this number, in analogy with terminology in the theory of domination in graphs, the domatic number, and denoted by ς (M), for a given finite matroid M. Any collection of ς (M) pairwise disjoint domsets in M is then called a domatic collection of M. If these collections turn out to be partitions of the respective types of the ground set of the matroids. In general, irrespective of whether the given matroid M is finite or infinite, we call M dompartitionable if its ground set V (M) can be partitioned into its domsets. Further, M is finitely dompartite if there exists a partition of V(M) into finitely many domsets of M, called a finite dompartition of M.

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IV. CONCLUSION

Theorem 3.2. In a locally finite matroid M = (V, M), any minimal domset is either a base or it contains a circuit.

Proof: Let D be any minimal domset of M and suppose it is not a base. If D is independent, since by assumption it is not maximal, it follows that there exists a base P that properly contains D. Since cl(D) = V, we see that for every v ∈ V−D, there must exist a nonempty subset A of D such that A ∪ {v} ∈ C_M. Therefore, since P−D ⊂ V−D, there exists, in particular, a vertex u ∈ P −D such that B ∪{u} ∈ C_M for some nonempty subset B of D. But this is contradictory to the independence of P. Thus, D cannot be independent and hence must be dependent. Since M is locally finite, D must contain a circuit.

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DECLARATION STATEMENT

Some of the cited references are older and are noted explicitly as [2]. However, these works remain significant for the current study, as they are pioneering in their fields.

After aggregating input from all authors, I must verify the accuracy of the following information as the article's author.

- **Conflicts of Interest:** Based on my understanding, this article has no conflicts of interest.
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- **Data Access Statement and Material Availability:** The adequate resources of this article are publicly accessible.
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