



Transformative Quantum Geometry and the Concept of Discrete Quantum Spaces of the Universe

Nishant Sahdev, Chinmoy Bhattacharya

Abstract: The conventional geometry calculates the areas and volumes of different geometric figures in the macroscopic domain by considering area as the product of length and breadth, and volume as the product of length, breadth, and height. However, the literature provides no explanation for why area and volume are calculated as products of spatial dimensions. The unit-square concept of area and volume is also biased in this manner. So, this research has been conducted to address these conceptual gaps; to do so, we introduced a new framework, i.e., Transformative Quantum Geometry (TQG). In this research article, the areas and volumes of geometric entities such as squares, circles, cylinders, and spheres are calculated in terms of Planck's length. Planck's length is a universally accepted, the smallest possible length of the universe in direct space. A model of transformative quantum geometry is being developed and presented in this article, where the areas and volumes are being considered to be the summation of overlapped distances only from x and y directions for 2D geometric entities like squares and circles, and the summation of distances for the 3D geometric entities from x, y, and z directions. The mathematical formulas for the areas and volumes of squares, circles, cubes, cylinders, spheres, etc., are first developed in the microscopic domain and then extended to the macroscopic domain. This work would provide a new interpretation of measurements.

Keywords: Transformative Quantum Geometry; Discrete Space; Planck-Length Geometry; Area-Volume Quantisation; Non-Euclidean Geometric Construction

Nomenclature:

TQG: Transformative Quantum Geometry

lp: Planck length

I. INTRODUCTION

The area of a square or rectangle is conventionally calculated by multiplying length and breadth, and the volume by multiplying length, breadth, and height. There is no grounded concept explaining why the areas and volumes are being calculated in this manner. It is true that in ancient civilisations, people used this 'tool of multiplication' for land measurements.

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*Correspondence Author(s)

Nishant Sahdev, Department of Research & Development, Austin Paints & Chemicals Pvt. Ltd., Kolkata (West Bengal), India. Email ID: nishantsahdev.onco@gmail.com, ORCID ID: [0009-0007-2249-1006](https://orcid.org/0009-0007-2249-1006)

Dr. Chinmoy Bhattacharya*, Department of Research & Development, Austin Paints & Chemicals, Kolkata (West Bengal), India. Email ID: chinmoy00123@gmail.com, ORCID ID: [0000-0002-1962-0758](https://orcid.org/0000-0002-1962-0758)

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However, this method continued, and the root went very deep, securing a permanent place in the fields of mathematics, geometry, physics, engineering, chemistry, and many others. [10] [11].

The unit square concept is also not logically justified, since it multiplies the length and breadth (each being unity) to express the area as 1. Indeed, this concept has made the entire subject of mathematics, and other areas too, very clumsy by introducing so many dimensions, such as square meters, meters, ... up to meter^[n]. The lengths would obviously appear in different forms of power, but those do not inherently arise from squaring or cubing a length variable [11].

In the model of this article, for example, the power n does arise from a variable of length x as xⁿ in the mathematical expressions for the areas and volumes of numerous geometric entities, not from passing to the nth dimension. Still, it is shown that xⁿ arises out of the cumulative summation of x of the number of recurrences of the variable x.

This is shown below:

$$X^2 = X + X + X + X + \dots \dots \dots X \text{ times}$$

$$X^3 = X^2 + X^2 + X^2 + X^2 + \dots \dots \dots X \text{ times}$$

$$\dots \dots \dots$$

$$\dots \dots \dots$$

$$X^n = X^{(n-1)} + X^{(n-1)} + X^{(n-1)} + X^{(n-1)} + \dots \dots \dots X \text{ times}$$

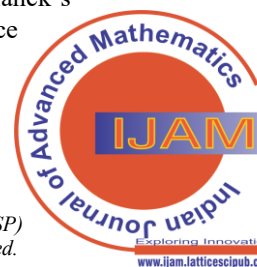
In this model, the area is considered as the summation of distances only. For example, for a square entity of L Planck's length, if some finite numbers of concentric squares are drawn [the length of the said concentric squares being progressively reduced by the smallest possible length of the universe, an unit of Planck's length (lp) from one concentric square to the next concentric square] within the outer periphery of the square, this will end at the center of the square as a geometrical point only. In this journey, the gap or distance between one concentric square and the next concentric square is retained to be a unit of Planck's length only. The 'area' of the square of Planck's length L would be [11].

A. Area of a square of L Planck's length

= [(sum of the circumferences of the outer square of L Planck's length plus the circumferences of the squares of Planck's length, (L-1), (L-2), (L-3), (L-4) {L-(L-1)}, (L-L) lengths respectively].

B. Volume of a cube of L Planck's length

= [(sum of the circumference lengths vis-a- vis the surface areas of the outer cube of L Planck's length plus the circumference lengths vis-a- vis the surface areas of the cubes of Planck's

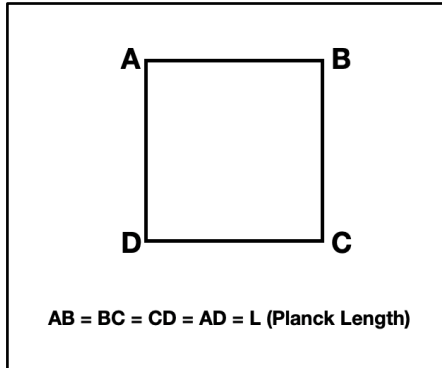


length, (L-1), (L-2), (L-3), (L-4) {L- (L-1)}, (L-L) lengths respectively].

In this model, 1, 2, 3, 4, 5... etc do stand for units of Planck's length, not as an arbitrary unit length 1 of the conventional geometry [11].

II. THE MATHEMATICAL FORMULA OF 'SQUARE'

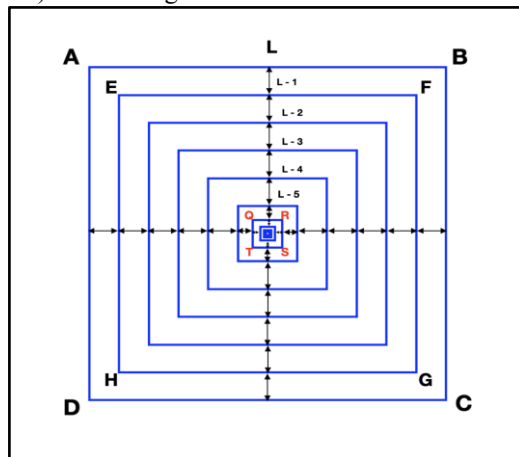
Let a square be considered, as shown in Fig.1; the length of each side is 'L' Planck length.



[Fig.1: A Square ABCD with Side Length L, measured in units of Planck Length, Where $L = n l_p$, where n is a Positive Integer, and l_p Denotes the Planck Length]

As explained in the introduction, the area and volumes represent only the total of sweeping and overlapping distances. So, for the square shown in Fig.1, we need to consider a finite number of small squares within it.

As shown in the Fig.2, the outermost square is made of L Planck lengths, and the inner squares have lengths decreasing in a manner, L to (L-1) to (L-2) to (L-3) to (L-4) to.... [(L-(L-1)) to (L-L) Planck length.



[Fig.2: Representation of a Square ABCD of Length L with its Concentric Square Areas]

The distance between two consecutive inner circles is only 1 Planck's length. In this manner, one reaches a point at the centre of the square, O. Since the area is the sum of overlapped distances, if Person-1 starts walking from Point A of the outer Square and walks through the four arms, AB, BC, CD, and DA in his first round of walking, another person, Person-2, would be standing at the centre point O only. Then, while Person-1

starts his 2nd round of walking through the 1st inner square EFGH, Person-2 will make his first round of walking through the innermost square QRST [11].

Person-1 will go on walking shorter and shorter distances as his round of walking becomes higher (round-1, round-2, round-3, round-4 ...), and on the contrary, Person-2 will walk longer and longer distances as his round of walking becomes higher. In this journey, the distances will overlap, and at the end, person-1 will reach the centre of the square, and person-2 will reach point A. However, the sum of the distances of the walk of several rounds will be the same for person-1 and person -2. If person 1 walks a total distance S, the second person will also walk the same distance S.

[To note here that in this manner, the area vis-à-vis the surface areas of both the faces (the outer face and the inner face) is considered.

In this transformative Quantum geometry model,

The area of the square ABCD would be

= [the sum of the distances walked by the person-1 and person -2, in an overlapped manner] = [S + S] = 2S (1)

Now, the mathematical expression of S in equation (1) regarding person -1 would be calculated.

Person-1 in 1st round walks a distance = $4 \times L = 4L$ (1a)

Person-1 in 2nd round walks a distance = $4 \times (L-1) = 4(L-1)$ (1b)

Person-1 in 3rd round walks a distance = $4 \times (L-2) = 4(L-2)$ (1c)

Person-1 in 4th round walks a distance = $4 \times (L-3) = 4(L-3)$ (1d)

Person-1 in 5th round walks a distance = $4 \times (L-4) = 4(L-4)$ (1e)

Person in his [L- (L-1)]th round walks a distance = $4 \times [L- (L-1)] = 4$ (1f)

Person in his Lth round walks a distance = $4 \times (L-L) = 0$ (1g)

So, summing up all the rounds of walk for the person-1 [equation (1a) to equation (1g)],

$S = [4L + 4(L-1) + 4(L-2) + 4(L-3) + 4(L-4) + \dots + 4 \times [L- (L-1)] + 0]$

$= [(4L) + (4L) + (4L) + (4L) + \dots L \text{ number of terms}] - 4 [1+2 +3 +4 + \dots (L-1) \text{ terms}]$

$= [4L \times L] - 4. (L-1)/2 [2 + (L-2).1]$

$= [4L^2] - 2L (L-1)$

$= 4L^2 - 2L^2$ (For a value of $L > 100$ Planck length, for example, 1 can be neglected since 1 represents a unit of Planck's length in this model]

$= 2L^2$

or $S = 2L^2$ (2)

The total distance walked by person-2 can be derived in the same manner as equation (1) was derived, but in reverse, and it will be S too.

Area of square ABCD of each side being L Planck length, is = $2 \times [2L^2] = 4L^2$ (3)

However, the macroscopic length of a square X will be an integral multiple of a unit of Planck's length.

So, $X = n. (\text{Planck's length})$, [where n is an integer] (4)

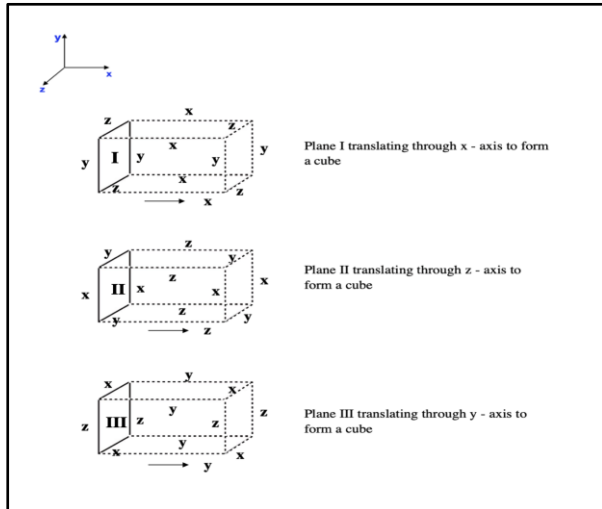
So, the generalized formula of the area of a square of length X, would be, Area of a square of length X = $4X^2$ (5)



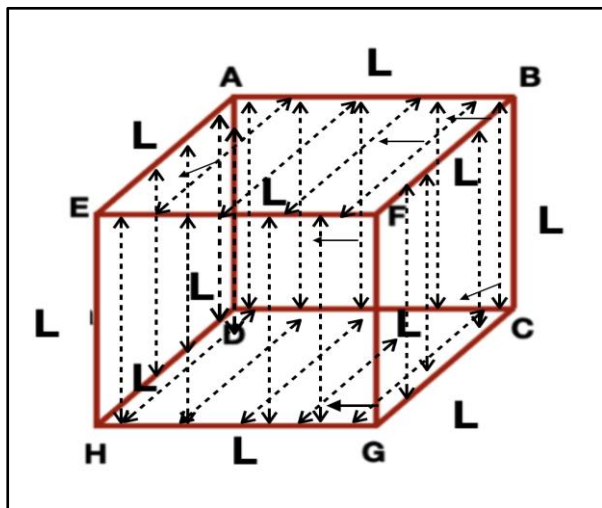
The generalized formula of the area of a rectangle of length and breadth X and Y, respectively, would be,
Area of a rectangle (length X and breadth Y) = $4XY$ (6)

III. THE MATHEMATICAL FORMULA OF THE VOLUME OF A CUBE

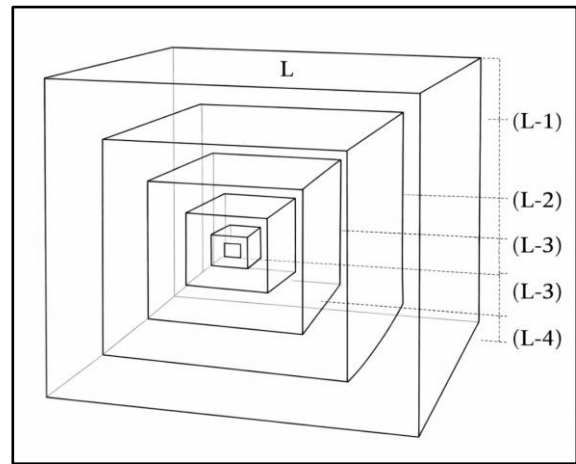
[Fig.3](#) through 5 break down exactly how this framework evaluates the volume of a cube (side length L, in Planck units) using a discrete, layer-by-layer construction. When, for example, a floor is being swept by a broom, every point of the floor has to be contacted by the surface of the broom.



[Fig.3: The Formation of a Cube of Length L, from the Translations of 3 Numbers of Squares Planes x-y, x-z and y-z Respectively to their Opposite Planes Respectively to form a Cube]



[Fig.4: The Sweeping Action of a thin wire (the Thickness of the wire Vanishingly Small) of Length L Through the Through Surface 1, Surface 2, Surface 3, Surface 4, Surface 5 and Surface 6]



[Fig.5: The Inner Cubes of Planck's Length, (L - 1), (L - 2), (L - 3), (L - 4).. Respectively Within the Outer Cube of Planck's Length L]

The above [Fig.3](#), [Fig.2](#) to [Fig.5](#) are self-explanatory. The volume of the Cube is being calculated now based on the following Figures.

Total volume of the cube = Sum of the distances travelled by the thin wire [(Plane 1 to 2 to 3 to 4 to 5 to 6) (7)]

Now the total distance of equation (7), (considering all the inner cubes having 6 numbers of faces each) and keeping in mind that the area [cumulative distances of a square figure of length L, as derived in this article, is $4L^2$, equation (5) above].
= $6[4L^2 + 4(L-1)^2 + 4(L-2)^2 + 4(L-3)^2 + \dots + 4(L-L)^2]$

$$= 24 [L^2 + (L^2 - 2L + 1) + (L^2 - 4L + 4) + (L^2 - 6L + 9) + \dots + 0]$$

$$= 24 [(L^2 + L^2 + L^2 + \dots L \text{ terms}) - L \{(2 + 4 + 6 + 8 + \dots (L-1) \text{ terms})\} + \{(1^2 + 2^2 + 3^2 + 4^2 + \dots (L-1) \text{ terms})\}]$$

$$= 24 [L^3 - L\{(L-1)/2(2 + (L-2))\} + \{(L-1)(L-1+1)(2L-2+1)/6\}]$$

$$= 24 [L^3 - L^2(L-1) + L(L-1)(2L-1)/6]$$

$$= 24 [L^3 - L^3 + L^3/3] \text{ [For } L > 100, 1 \text{ can be neglected]}$$

$$= (8L^3) \quad (8)$$

Now, as shown in [Fig.3](#), the original cube of length L is being formed from the fusion of 3 cubes of length L from the x-y, x-z and y-z directions, and so the total volume of the cube would be,

$$\text{The volume of a cube of length } L = 3 \times (8L^3) = 24L^3 \quad (9)$$

The total surface area of a cube having 6 faces (area of each face is $4L^2$) = $24L^2$

However, in this model, when the area vis-à-vis the surface area of a square (or rectangle) is considered (of length L), the $4L^2$ of equation (5) stands for the two surfaces of the square. One is the front side (outer surface)

of area $2L^2$, and the other one (inner surface) is $2L^2$ too. So, proceeding with this logic,

$$\text{The total outer surface area of a cube of length } L = 6 \times 2L^2 = 12L^2 \quad (10)$$

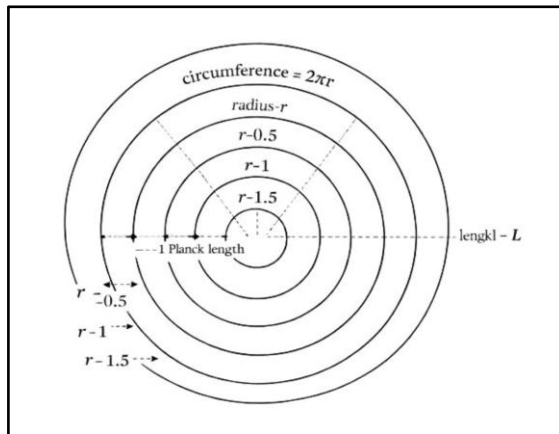
$$\text{The total inner surface area of a cube of length } L = 6 \times 2L^2 = 12L^2 \quad (11)$$

The volume of a rectangular cube having the length, breadth and height being X, Y and Z respectively, would be $= 24XYZ$ (12)

IV. THE MATHEMATICAL FORMULA OF THE AREA OF A CIRCLE

The walking of two persons (Person -1 and Person-2) through the circumference of the outer circle (radius r , circumference length $2\pi r$) and the concentric circles [radius r , $r-0.5$, $r-1$, $r-1.5$, ... Planck's length and circumference of $\pi(r-i/2)$ is shown in Fig.6.

The diameter of the inner circles decreases by 1 Planck length consecutively, and hence the radii do decrease by 0.5 Planck lengths in each step. The linear distance between the two consecutive concentric circles is 1 Planck length [9].



[Fig.6: Representation of a Circle of Radius r Planck's Length with its Concentric Inner Circles]

$$\begin{aligned} \text{The total distance walked by the 1}^{\text{st}} \text{ person} &= [2\pi r + 2\pi(r-0.5) + 2\pi(r-1) + 2\pi(r-1.5) + 2\pi(r-2) + \dots + 2\pi\{r - (r-1)/2\} + 2\pi(r-r)] \\ &= [(2\pi r + 2\pi r + 2\pi r + \dots 2r \text{ times}) - \pi\{1 + 2 + 3 + 4 + \dots + (2r-1)\} \text{ terms}] \\ &= [4\pi r^2 - \pi(2r-1)/2\{2 + 2r - 2\}] \\ &= [4\pi r^2 - 2\pi(2r)/2(2r)] \text{ [for } L \text{ value} > 100, 1 \text{ can be neglected]} \\ &= [4\pi r^2 - 2\pi r^2] \\ &= 2\pi r^2 \end{aligned} \quad (13)$$

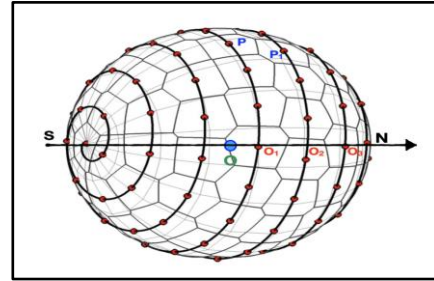
Now, considering the inner surface area and the outer surface area, both as being done in the case of a square in this article, The area of a circle of radius r = (inner surface area + outer surface area)

$$\begin{aligned} &= (2\pi r^2 + 2\pi r^2) \\ &= 4\pi r^2 \end{aligned} \quad (14)$$

V. MATHEMATICAL FORMULA FOR THE VOLUME OF A SPHERE

Fig.7 shows how a sphere forms [radius $= r$ (Planck's length)] regarding transformative geometry in the non- Euclidean space. As shown in the said Fig.7, from the centre (O) of the vertical equatorial circle of the sphere, a perfectly horizontal line, ON, is drawn (on the right-hand side) whose length is r only.

Now, this length is divided into r Planck lengths. The equatorial circle (marked as 1 in the Figure) has a diameter of $2r$, and the next circle, 2, is drawn at a horizontal distance of 1 Planck's length from the centre O of the first circle (whose centre is O1) with diameter $= (2r-1)$ Planck's length.



[Fig.7: Translation Begins at the Outer Surface of the Sphere and Proceeds Successively Through All Concentric Circular Layers]

The next circle is being drawn at a horizontal distance of 2 Planck's lengths from the point O with diameter $(2r-2)$. This drawing of circles is continued in this manner until it reaches the point N, which is a point only. (Divided into $2r$ numbers of Planck's length. This process of drawing circles is repeated on the left-hand side of the equatorial circle 1, until it reaches the point S. In this manner, the entire skeleton of the sphere is drawn.

Now, a point P1, for example, as shown in Fig.7, starts translating from the outer surface of the sphere, travels along the entire set of concentric circles (see Fig.6), and reaches the centre O of circle 1. Then, from O, it passes to a parallel point P2 (to point P1) and travels through all the concentric circles of circle 2; then it passes to circle 3 and travels through all the concentric circles of circle 3.

This translation of the point continues until it reaches point N. If the total path travelled by point Pi is SN and the total distance travelled by another point identically is SS, then the total distance travelled by the two points is, [9]

$$\begin{aligned} \text{Total distance travelled by the points on the right-hand side and the left-hand side of the equatorial circle would be} &= (S_N + S_S) = 2S_N \text{ [since } S_N = S_S\text{]} \quad (15) \\ S_N &= 4\pi r^2 + 4\pi(r-0.5)^2 + 4\pi(r-1)^2 + 4\pi(r-1.5)^2 + 4\pi(r-2)^2 + 4\pi(r-2.5)^2 + \dots 4\pi\{r - (r-0.5)\}^2 + 4\pi(r-r)^2 \\ &= (4\pi r^2 + 4\pi r^2 + 4\pi r^2 + 4\pi r^2 \dots 2r \text{ times}) - 4\pi\{1 + 2 + 3 + 4 + \dots + (2r-1) \text{ terms}\} + \{0.25 + 1 + 2.25 + 4 \dots (2r-1) \text{ terms}\} \end{aligned}$$



$$\begin{aligned}
 &= (8\pi r^3) - 4\pi r x (2r-1)/2 \{2 + 2r-2\} + 0.25 \{(1^2 + 2^2 + 3^2 + 4^2 \\
 &+ \dots (2r-1)\} \text{ terms} \} \\
 &= (8\pi r^3) - 4\pi r x (2r-1)/2 \{2r\} + 0.25 (2r-1) (2r) (4r-1)/6 \\
 &= (8\pi r^3) - (8\pi r^3) + 4/3 (\pi r^3) \\
 &= 4/3 (\pi r^3) \quad (16)
 \end{aligned}$$

Detailed Derivation of the Series S_N

We start with the given series: [equation (16)]

$$S_N = 4\pi r^2 + 4\pi(r-0.5)^2 + 4\pi(r-1)^2 + 4\pi(r-1.5)^2 + \dots + 4\pi(0.5)^2 + 4\pi(0)^2$$

Step 1: Factor out the constant 4π

$$S_N = 4\pi [r^2 + (r-0.5)^2 + (r-1)^2 + (r-1.5)^2 + \dots + 0^2] \quad (16a)$$

Step 2: Express the series using an index

The radius decreases in half-unit increments. Let the k_{th} term be:

$$r_k = r - k/2 \quad (16b)$$

The last term occurs when $r - k/2 = 0 \Rightarrow k = 2r$

So, the sum becomes:

$$S_N = 4\pi \sum_{k=0}^{2r} (r^2 - rk + k^2/4) \quad (16c)$$

Step 3: Expand the square

$$(r - k/2)^2 = r^2 - rk + k^2/4 \quad (16d)$$

Substitute into the sum:

$$S_N = 4\pi \sum [r^2 - rk + k^2/4] \quad (16e)$$

Using linearity of summation:

$$S_N = 4\pi \sum_{k=0}^{2r} [r^2 - rk + (1/4)k^2] \quad (16f)$$

Step 4: Evaluate the standard sums

$$\sum_{k=0}^{2r} r^2 = (2r+1) r^2 \quad (16g)$$

$$\sum_{k=0}^{2r} k = [(2r)(2r+1)]/2 = r(2r+1) \quad (16h)$$

$$\sum_{k=0}^{2r} k^2 = [(2r)(2r+1)(4r+1)]/6 \quad (16i)$$

Step 5: Substitute back

$$S_N = 4\pi [(2r+1)r^2 - r^2(2r+1) + (1/4)(2r)(2r+1)(4r+1)/6] \quad (16j)$$

The first two terms cancel exactly.

$$\text{Step 6: Final simplification } S_N = 4\pi (2r)(2r+1)(4r+1)/24 \quad (16k)$$

Expanding:

$$= 4\pi [(2/3)r^3 + (1/2)r^2 + (1/12)r] \quad (16l)$$

Final Result:

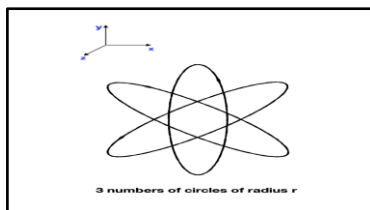
$$S_N = (8\pi/3)r^3 + 2\pi r^2 + (\pi/3)r \quad (16m)$$

Now the total distance travelled by the two points as per equation (15) above would be,

Total distance travelled by the points on the right-hand side and the left-hand side of the equatorial circle would be $(S_N + S_S)$

$$= 2S_N = (8/3)(\pi r^3) \quad (17)$$

Now, as shown in Fig.8 below, the total distance must be computed from the x-, y-, and z-directions. So, the total distance, as determined from equation (17) in the x direction, will overlap with the total distances in the y and z directions as well.



[Fig.8: The Positioning of the Equator Circle of a Sphere of Radius r from x, y and z Directions]

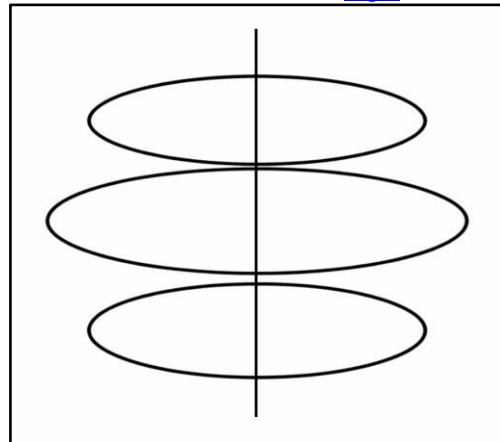
So, the total overlapped distances from the x, y and z directions would be,

$$\text{Total overlapped distance} = 3 \times 8\pi [(2/3)r^3 + (1/2)r^2 + (1/12)r] = [8\pi r^3 + 12\pi r^2 + 2\pi r] \quad (18)$$

Now, if in equation (16j) for higher values of r , the terms $(2r+1)$ and $(4r+1)$ could be considered as $2r$ and $4r$ respectively, and then equation (16m) reduces to,

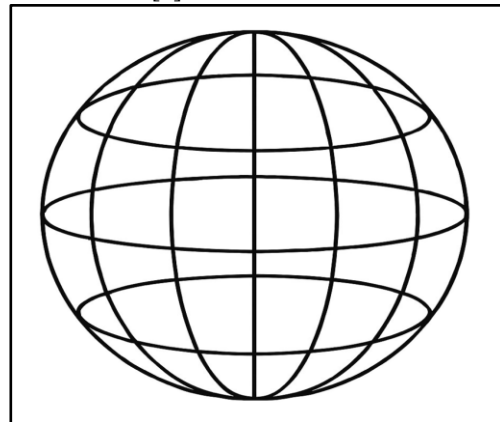
$$S_N = (8\pi/3) r^3 \quad (18a)$$

Considering the overlapping of distances as described in equation (18), the resulting geometric skeleton of a sphere of radius r assumes the form illustrated in Fig.9.



[Fig.9: The Look or the Shape of the Skeleton of a Sphere of Radius r]

As evident from Fig.9, the outer periphery of the sphere is not yet fully formed at this stage of the construction. Fig.10 illustrates the formation of the complete outer periphery through a rotation of the skeletal framework shown in Fig.9 by an angle of π radians [9].



[Fig.10: The Evolution of the Outer Frame of the Skeleton Structure of a 3D Sphere Upon the Rotation of the Frame by π Radian]

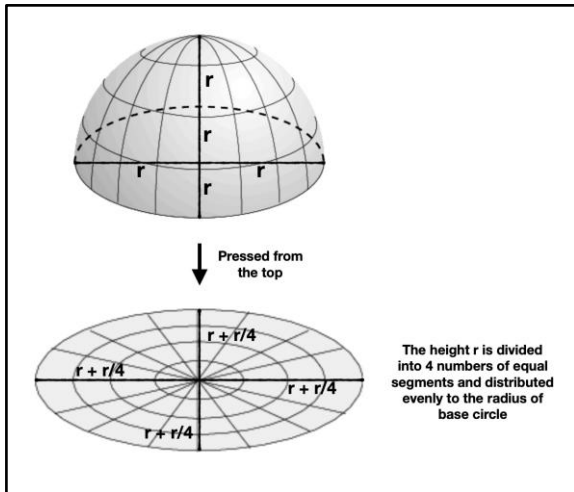
Consequently, the total distance derived in equation (18) is multiplied by a factor of π to obtain the actual total distance, since the rotation of the skeletal structure through π radians generates complete circular contributions from all constituent paths.

So, the volume of a sphere of radius r (as per the current model) would be

$$= [(8\pi r^3) \times \pi] = 8\pi^2 r^3 \quad (19)$$

The surface area of a sphere is now derived by invoking the principle of conservation of area within the framework of transformative geometry. As illustrated in Fig.11, a hemisphere of radius r is conceptually compressed from the top onto a flat surface and merged with that surface. Under this transformation, the hemispherical surface is mapped onto a two-dimensional circular region.

Because the hemisphere's height equals r , compressing it forces this vertical dimension to spread evenly across the circular base. Fig.11 maps out exactly how this works: the vertical height is reallocated symmetrically outward along the base's two longitudinal and two lateral axes. Crucially, this specific outward flattening perfectly preserves the total surface area.



[Fig.11: The Transformation of a Hemisphere of Radius r to a Circle Upon Pressing it from the top on a Flat Surface and Merging with the Same]

Radius of the transformed circle = [original radius of the base circle (r) + contribution from the height ($r/4$) = $(5r/4)$ (20)

Area of the transformed circle of the hemisphere would be = $4\pi (5r/4)^2 = 6.25\pi r^2$ (21)

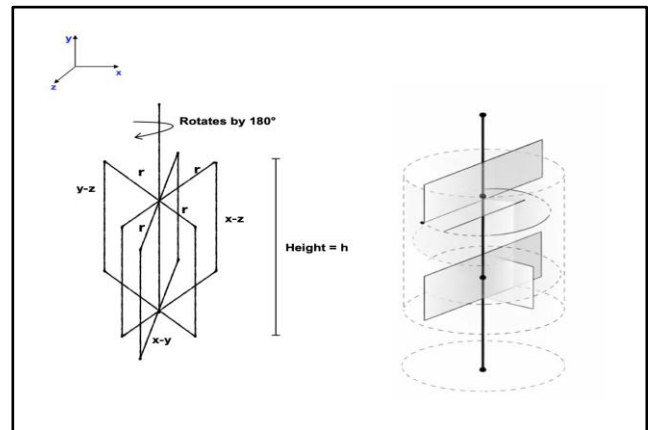
Area of the transformed circle of another hemisphere would be = $4\pi (5r/4)^2 = 6.25\pi r^2$ (22)

The total surface area of a sphere of radius r would be = $2 \times 6.25\pi r^2 = 12.5\pi r^2$ (23)

VI. THE MATHEMATICAL FORMULA OF THE VOLUME OF A CYLINDER OF RADIUS R

A cylinder of radius r is basically formed from the stacking of circles of radius r one after the other up to a certain height h . However, 2D circles have no thickness, and ideally, even an infinite stack of circles will not produce any height. The best

model of a cylinder is obtained by rotating a rectangle (of length $2r$ and breadth h) about its vertical axis by 180 degrees to form a cylinder of radius r and height h , as shown in Fig.12 below.



[Fig.12: The Rotation of Three Rectangles of Length $2r$ and Breadth h on Their Vertical Axes to form a Cylinder of Radius r and Height h]

The volume of the cylinder would be the sum of the overlapped distances of a point translating through the outer circles and the concentric circles through the x - y , x - z , and y - z directions.

[Unlike the case of a sphere, in the case of a cylinder, the Euclidean geometry of a 2D square is being transformed to a non-Euclidean geometry through the rotation of the square around its vertical axis. On the contrary, in the case of a sphere, a non-Euclidean 2D circle rotates on its vertical axis, retaining the non-Euclidean geometry] [7].

The height h Planck's length is being divided into h numbers of equal Planck's lengths, and through each of the h numbers of points a circle passes. So, the total distance a point would move is the sum of the distances along the circular paths.

As derived in this article, for a circle of radius r , the area is $2\pi r^2$ whenever one of the directions among x , y , and z is being considered [6].

So, the total distance travelled by a point through the x direction = $2\pi r^2 h$

Now, considering the x - y , y - z and x - z directions, the volume of a cylinder of radius r and height h would be,

$$= 3 \times 2\pi r^2 h = 6\pi r^2 h \quad (26)$$

Now the surface area of a cylinder of radius r and height h would be, = [(surface area or area of a rectangle of length $2\pi r$ and breadth h) + (area of two circles of radius r each at the two ends of the cylinder)

$$\begin{aligned} &= [(4 \times 2\pi r \times h) + (2 \times 4\pi r^2)] \\ &= [8\pi r h + 8\pi r^2] \\ &= 8\pi (h + r) \end{aligned} \quad (27)$$



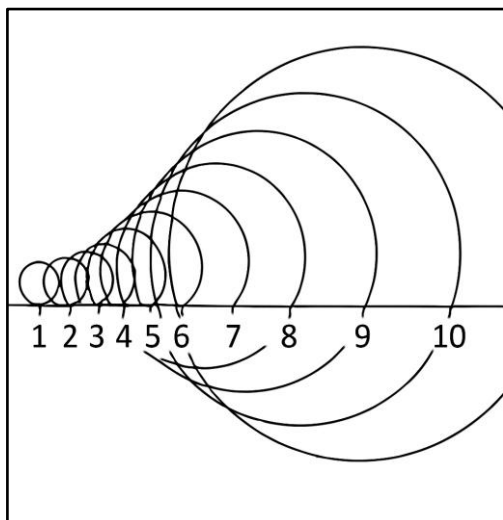
Table I: Scale Dependence of Discretized Geometric Growth. Percentage Increases in Area and Volume for a unit Planck-length Increment in the Parameters l or r Are Shown for Square, Circle, Cube, and Sphere Geometries, Comparing the Microscopic Regime ($l, r = 1, 2$) with the Macroscopic Regime ($l, r = 1000, 1001$), Based on the Discretised Formulations Developed in this Work

Geometric Entity	Core Mathematical Equation of the Geometric Entity (Considering the Discretization Parameters Derived in the Article)	Value of L (Microscopic Quantum State)	Value of $(L + 1)$ (Microscopic Quantum State)	Value of Area / Volume (Microscopic Quantum State) Value for $L = 1$	Value of Area / Volume (Microscopic Quantum State) Value for $L = 2$	Percent Increase of Area / Volume (Microscopic Quantum State)	Value of L (Macroscopic Quantum State)	Value of $(L + 1)$ (Macroscopic Quantum State)	Value of Area / Volume (Macroscopic Quantum State) Value for $L = 1000$	Value of Area / Volume (Macroscopic Quantum State) Value for $L = 1001$	Percent Increase of Area/Vol ume (Macroscopic Quantum State)
Square, Area	$[(2L^2 + 2L)]$	1	2	4	12	200%	1000	1001	2002000	2006004	0.20%
Cube, Volume	$4[(2L^3 + 6L^2 - 3L + 1)]$	1	2	24	140	483.30%	1000	1001	$8.024E+09$	$8.048E+09$	0.30%
Circle, Area	$[(2\pi r^2 + \pi r)]$	1	2	9.42	31.41	234%	1000	1001	6285183	6297194	0.20%
Sphere, Volume	$[(8\pi/3)r^3 + 2\pi r^2 + (\pi/3)r]$	1	2	15.7	94.2	500%	1000	1001	$8.382E+09$	$8.41E+09$	0.30%

Table II: Relative Contribution of Discretization Parameters to Geometric Areas and Volumes Across Microscopic and Macroscopic Spatial Domains

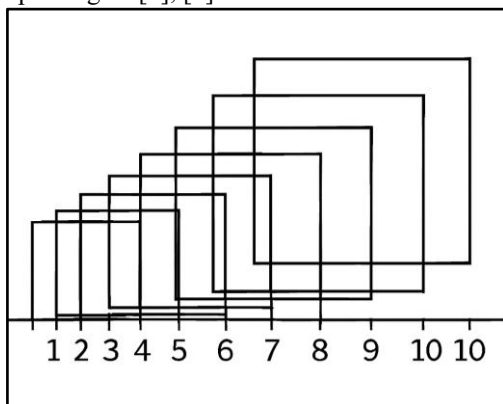
Geometric Entity	Core Mathematical Equation of the Geometric Entity (Considering the Discretization Parameters Derived in the Article)	Value of L (Microscopic Quantum State)	Value of A_S or V_C or C_A or V_S	Value of Contribution of Discretization Term	% Contribution of the Discretization Term on the Total Value of Area or Volume (Microscopic Quantum State)	Value of L (Macroscopic Quantum State)	Value of A_S or V_C or C_A or V_S	Value of Contribution of Discretization Term	% Contribution of the Discretization Term on the Total Value of Area or Volume (Macroscopic Quantum State)
Square, Area	$A_S = [(2L^2 + 2L)]$ Discretization terms = $(2L)$	1	2	1	50%	1000	2002000	2000	0.1%
Cube, Volume	$V_C = 4[(2L^3 + 6L^2 - 3L + 1)]$ Discretization term = $= 4[(6L^2 - 3L + 1)]$	1	24	16	66%	1000	8023988004	23988004	0.3%
Circle, Area	$C_A = [(2\pi r^2 + \pi r)]$ Discretization term = $[(\pi r)]$	1	9.42	6.29	33%	1000	62839	3141	0.04%
Sphere, Volume	$V_S = [(8\pi/3)r^3 + 2\pi r^2 + (\pi/3)r]$ Discretization term = $[2\pi r^2 + (\pi/3)r]$	1	15.7	7.3	47%	100	8382300	6283940	0.75%

If we place [Table I](#) alongside [Table II](#), a clear physical trend emerges: as we scale up from the microscopic domain, these quantized geometric measures naturally recover their classical continuum limits [8] [1], [4], [5].



[Fig.13: Overlapping Pattern of Equidistant Circles Arranged Along a Straight Line]

[Fig.13](#) depicts the geometric transition. When the radius is small, adjacent circles overlap, maintaining a discrete, quantized structure. But as the radius scales up, this overlap compounds. The individual boundaries begin to blur and merge, forms an effectively continuous spatial region. This visualizes exactly how a smooth macroscopic space emerges directly from microscopic origins [2], [3].



[Fig.14: Overlapping Pattern of Equidistant Squares Arranged Along a Straight Line]

As shown in [Fig.14](#), for smaller side lengths, adjacent squares show limited overlap, maintaining a discrete structure. As the side length increases, the individual squares merge into a continuous spatial region, showing the emergence of a space continuum at larger length scales [6].

VII. CONCLUSION

This research article presents conventional abstract measurements of the areas and volumes of various geometric figures by multiplying length by breadth to obtain area and length, breadth, and height to obtain volume. This unit square or unit cube concept of conventional geometry or mathematics does not have any tripartite physical significance regarding physics, mathematics, and topology. While physicists around the globe are looking for a theory that will show the pathway to express the magnitude of any physical variable in the universe

in terms of ‘fundamental lengths’ only, no one has yet shown that pathway. In this article, it has been strongly established that areas and volumes of any geometric entities could be obtained by the summation of ‘discrete and quantized length paths only of infinitesimally small gaps in space. The appearance of the square and cube of the lengths originates from the repetitive accumulation of lengths only, and not by any dimensional jumps. The area and volumes are to be expressed in terms of distance or meters only, and not in square meters or cubic meters, depending on the scale used.

Another important phenomenon shown in this research article is that, at the microscopic level, the physical variables (such as areas and volumes, as shown here) exhibit sharp, observable gaps. Still, as one passes to the macroscopic level, a continuum develops, in which the gaps between the different higher-order areas or volumes are progressively merged.

The concept presented in this article has very broad implications for the standard modes of operation of Classical Mathematics, the Standard Model of Physics, and Physical Chemistry, and, in the coming days, the conventional framework of these branches of science will need a drastic turnaround.

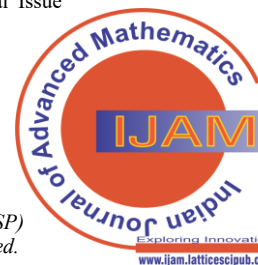
DECLARATION STATEMENT

I must verify the accuracy of the following information as the article's author.

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REFERENCES

1. Yang, B.-J. (2025). From Berry curvature to quantum metric: A new era of quantum geometry metrology for Bloch electrons in solids. arXiv:2512.24553. <https://arxiv.org/abs/2512.24553>
2. Yu, J., Bernevig, B. A., Queiroz, R., Rossi, E., Törmä, P., & Yang, B.-J. (2025). Quantum geometry in quantum materials. npj Quantum Materials, Nature Portfolio. <https://www.nature.com/articles/s41535-025-00801-3>
3. Liang, S.-D., Harko, T., & Lake, M. J. (Eds.). (2023). New advances in quantum geometry. MDPI Special Issue Collection. <https://www.mdpi.com/2624-8174/5/3/688>
4. Törmä, P. (2023). Where can quantum geometry lead us?





- Physical Review Letters, 131, 240001. (Conceptual essay on the transformative role of quantum geometry in superconductivity and flat-band systems.)
<https://arxiv.org/abs/2312.11516>
5. Kang, M., et al. (2024). Quantum geometry in solids measured using photoemitted electrons. Nature Physics. (First direct experimental measurement of quantum geometry in quantum materials.)
<https://api.semanticscholar.org/CorpusID:274290529>
 6. Godin C, Boudon F. Riemannian L-systems: modelling growing forms in curved spaces. Quant Plant Biol. 2025 Jul 1;6:e38. PMID: 41445926; PMCID: PMC12722102. DOI: <http://doi.org/10.1017/qpb.2025.10014>
 7. Aguirre-López, M.A.; Hueyotl-Zahuantitla, F.; Martínez-Vázquez, P. Advances in Numerical Data Visualization of Flow around a Square Cylinder. Symmetry 2024, 16, 32.
DOI: <https://doi.org/10.3390/sym16010032>
 8. Li, B. Why Geometry Should Not Be Quantized: A Causal-Medium Unification of Gravity and Quantum Mechanics. Quantum Rep. 2026, 8, 2.
DOI: <https://doi.org/10.3390/quantum8010002>
 9. Bunge, M. Inverse Problems. Found Sci 24, 483–525 (2019).
DOI: <https://doi.org/10.1007/s10699-018-09577-1>
 10. Escors, D., & Kochan, G. (2022). Covariant Space-Time Line Elements in the Friedmann–Lemaître–Robertson–Walker Geometry. Axioms, 11(310).
DOI: <https://doi.org/10.3390/axioms11070310>
 11. Shah, N. A., Ashraf, S. S. Z., Shaikh, A., Yamin, Y., Sahoo, P. K., Bhat, A., Lone, S. A., Faizal, M., & Ahsan, M. A. H. (2023). Derivation of the deformed Heisenberg algebra from discrete spacetime. EPL, 149 (2025) 40001. DOI: <https://doi.org/10.1209/0295-5075/ada79d>

AUTHOR'S PROFILE



Nishant Sahdev is a distinguished researcher. His work, based on quantum gravity, cosmology, and dark matter, contributes to the development of a unified quantum gravity theory and an innovative space quantisation model.



Dr. Chinmoy Bhattacharya earned his PhD in Polymer Physics in 1988 and completed postdoctoral research on liquid-crystal polymers at Laval University in Canada. Returning to India in 1991, he joined ICI India Ltd. and later founded his own paint company. He is a former chairman of the Indian Paint Association and a leading figure in India's paint industry. Bhattacharya's research spans quantum gravity, cosmology, and dark matter, culminating in a unified quantum gravity theory and a novel space quantization model. As a guest faculty member at the University of Calcutta, he teaches Colour Physics, Polymer Physics, and Rheology of Coatings to postgraduate students, continuing to inspire the scientific community.

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