



Analysis of Redundant System with Facility for Repair/Replacement of Substandard Unit

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Abstract: This paper analyses a system with two non-identical units, one of higher quality than the other. The inferior-quality unit can be repaired or replaced with a similar one. These systems occur in the electronics industry, hospitals, and other settings. A general time distribution is used for repair times. A technician for repair/replacement is consistently available with the system. This analysis evaluates the system's reliability through various measures. Practical examples include assessing system performance, analysing its effectiveness, and understanding its behaviour under different circumstances. Reliability measures are calculated to evaluate the system's performance. An exponential time distribution is used for failure; for other time distributions, reliability measures that assess system effectiveness are obtained. The average time the system operates before experiencing a failure, the time the system is operational and available for use, how often the repairman needs to visit the system for maintenance actions, the period when the repairman is busy due to maintenance tasks, and the profit generated by the system are calculated using mathematical techniques that model the system's behaviour, using the Semi-Markov procedure and the method of regenerative points. MATLAB is used for numerical and graphical analysis. By combining mathematical models, statistical distributions, and computational tools like MATLAB, this analysis provides insights into the system's behaviour and effectiveness.

Keywords: Mean Failure Time, Uptime, Busy Period, Substandard, Regenerative Point Method.

I. INTRODUCTION

Various studies and researchers have explored different aspects of reliability analysis involving replacement and repair policies in engineering and system maintenance. Goel et al. [2] (1984), Murari et al. [5] (1985), and Tuteja et al. [8] (1992) examined 2-unit systems that can undergo 2 or 3 repairs. These studies likely focused on optimizing repair and replacement strategies for such systems. Singh et al. [12] (1987) and Goel et al. [1] (1990) introduced replacement, inspection, and corrective maintenance, discussing maintenance actions based on inspection outcomes. Nakagawa et al. [6] (1975) and Mokkadis et al. [4] (1997) analysed systems with two non-identical units and repair facilities to optimise repair processes and replacement decisions. Render et al [7]. (1992) discussed a model with a substandard standby unit

and its replacement to manage standby units effectively and ensure system reliability. Singh et al. [11] (2001) studied a two-unit model with different repair strategies to assess the impact of repair methods on system performance. Sindhu and Gupta [10] (2002) explored systems using a visiting repairman facility. They focused on optimising the Scheduling of repairman visits for maintenance actions. Sharma and Kaur [9] (2016) conducted a profit analysis of a system with two compressor units and an essential generator. They investigated how different maintenance approaches affect system profitability. Kumar and Kumar [3] (2018) compared hardware and software models for failure and recovery coverage. Savita et al. (2021) explored reliability analysis of a system with repair/replacement of a cold standby unit and its impact on system reliability and downtime.

This chapter describes reliability analysis involving two types of units: a superior quality unit and a substandard unit. The substandard unit is subject to repair or replacement upon failure, with replacement using a unit of the same substandard quality. Meanwhile, the superior-quality unit remains operable while the substandard unit is replaced.

Reliability models that involve repair and replacement policies for substandard units are crucial in various industries to ensure systems or products continue to function. These models help optimise maintenance strategies, reduce downtime, and manage costs effectively.

II. SYSTEM DESCRIPTION

S : Set of states that are regenerative $\{S_0, \text{non-regenerative } \{capS_3, S_4\}$

$\bar{S}$: Set of states that are non-regenerative $\{S_2, S_5\}$

θ : 1st unit failure rate

ϕ : 2nd unit failure rate

$g_1(t), G_1(t)$: 1st Unit PDF and CDF of repair time

$g_2(t), G_2(t)$: 2nd unit PDF and CDF of repair time

p : Repairable Probability of 2nd unit if it fails

q : Non-repairable probability of 2nd unit when fails

$h(t), H(t)$: pdf and cdf of 2nd unit Replacement rate

P_{mn} : Probability of transition from state S_m to S_n

$P_{m,n}^{(k)}$: Probability of transition from state S_m to S_n via S_k

μ_l : Average sojourn time in S_l state

$\odot$ / S : Laplace Convolution / Laplace Stiejeles Convolution

$\wedge$: Laplace Transformation/Laplace Stiejeles Transformation

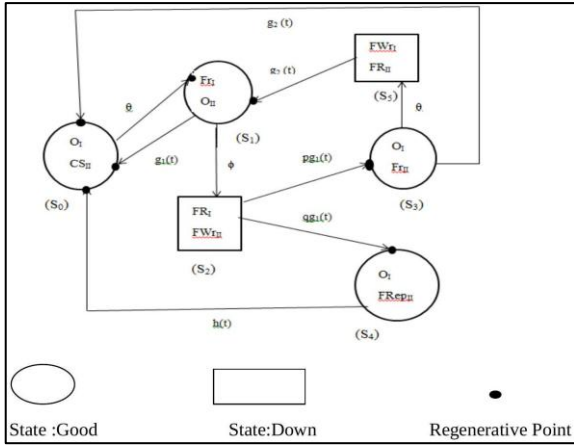
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III. TRANSITION DIAGRAM



[Fig.1: Transition Diagram]

A. Symbols Used

O_I/O_{II} : First/Second unit is operative and
 CS_{II} : Second unit is cold standby
 Fr_I/Fr_{II} : First/Second unit has failed and is under repair
 FR_R/FR_{II} : First/Second unit has failed, repairing from its previous state and
 FWr_R/FWr_{II} : First/Second unit is in failed mode, waiting to be repaired
 $Frep_{II}$: Second unit is failed, under replacement

IV. TRANSITION PROBABILITIES AND MEAN SOJOURN TIME

$$Q_{0,1}(t) = \int_0^t \theta e^{-\theta u} du \quad Q_{1,0}(t) = \int_0^t e^{-\phi u} g_1(u) du$$

$$Q_{1,2}(t) = \int_0^t \phi e^{-\phi u} \overline{G_1(u)} du \quad Q_{3,0}(t) = \int_0^t e^{-\theta u} g_2(u) du$$

$$Q_{3,5}(t) = \int_0^t \theta e^{-\theta u} \overline{G_2(u)} du \quad Q_{4,0}(t) = \int_0^t h(u) du \quad (1-6)$$

$$q_{0,1}(t) = \theta e^{-(\theta)t} \quad P_{0,1} = 1$$

$$q_{1,0}(t) = e^{-(\phi)t} g_1(t) \quad P_{1,0} = g_1^*(\phi)$$

$$q_{1,2}(t) = \phi e^{-(\phi)t} \overline{G_1(t)} \quad P_{1,2} = [1 - g_1^*(\phi)]$$

$$q_{3,0}(t) = [e^{-(\theta)t} g_2(t)] \quad P_{2,0} = g_2^*(\theta)$$

$$q_{3,5}(t) = [\theta e^{-(\theta)t} \overline{G_2(t)}] \quad P_{3,5} = [1 - g_2^*(\theta)] \quad (7-20)$$

$$P_{4,0} = [h^*(0) = 1] \quad P_{1,3}^{(2)} = p(1 - g_1^*(\phi))$$

$$P_{1,4}^{(2)} = q(1 - g_1^*(\phi)) \quad P_{3,1}^{(5)} = [1 - g_2^*(\theta)]$$

From above, we get

$$P_{0,1} = 1 \quad P_{1,0} = 1 \quad P_{1,2} = 1 \quad P_{1,0} + P_{1,3}^{(2)} + P_{1,4}^{(2)} = 1 \quad P_{3,0} + P_{3,5} = 1$$

$$P_{3,0} + P_{3,1}^{(5)} = 1 \quad P_{4,0} = 1 \quad (21-26)$$

These equations ensure that the probabilities of all possible transitions from each state sum to 1, a common requirement in Markov chains and probabilistic models that ensures the system moves to a valid state with probability 1.

$$\mu_0 = \frac{1}{\theta} \quad \mu_1 = \frac{1 - g_1^*(\phi)}{\phi} \quad \mu_3 = \frac{1 - g_2^*(\theta)}{\theta} \quad \mu_4 = -[h^*(0)] \quad (27-30)$$

$$m_{0,1} = \mu_0 m_{1,0} + m_{1,2} = \mu_1 m_{1,0} + m_{1,3}^{(2)} + m_{1,4}^{(2)} = [-g_1^*(0) = K_1]$$

$$m_{3,0} + m_{3,5} = \mu_3 m_{3,0} + m_{3,1}^{(5)} = [-g_2^*(0) = K_2] \quad m_{4,0} = \mu_4 \quad (31-36)$$

B. Cost and Benefit Analysis

The expected profit is

$$P = C_0 A_0 - C_1 B_0 - C_2 V_0 - C_3 R P_0$$



where

$$C_0 =$$

Revenue per unit time for which the system is up

C_1 = Cost per unit repairman's busy time

C_2 = Cost of repairman's each visit

C_3 = Cost of each replacement

V. PARTICULAR CASE

For graphical representation, the particular case is considered.

$$g_1(t) = \delta e^{-\delta t}$$

$$g_2(t) = \lambda e^{-\lambda t}$$

$$h(t) = \alpha e^{-\alpha t}$$

Thus, we have

$$P_{0,1} = 1$$

$$P_{1,0} = \frac{\delta}{\delta + \phi}$$

$$P_{1,2} = \frac{\phi}{\delta + \phi}$$

$$P_{1,3}^{(2)} = p \frac{\phi}{\delta + \phi}$$

$$P_{1,4}^{(2)} = q \frac{\phi}{\delta + \phi}$$

$$P_{3,0} = \frac{\lambda}{\lambda + \theta}$$

$$P_{3,5} = P_{3,1}^{(5)} = \frac{\theta}{\lambda + \theta}$$

$$P_{4,0} = 1$$

$$\mu_0 = \frac{1}{\theta}$$

$$\mu_1 = \frac{1}{\delta + \phi}$$

$$\mu_3 = \frac{1}{\lambda + \theta}$$

$$\mu_4 = \frac{1}{\alpha}$$

$$K_1 = \frac{1}{\delta}$$

$$K_2 = \frac{1}{\lambda}$$

VI. NUMERICAL AND GRAPHICAL ANALYSIS

Consider values:

$$\theta = 0.11,$$

$$\phi = 0.11,$$

$$\delta = 0.41,$$

$$\lambda = 0.71,$$

$$\alpha = 0.41,$$

$$p=0.5,$$

$$q = 0.5$$

A. Mean (Average) Time to System Failure (MTSF)
 $(T_0) = 61$

Availability **$(A_0) = 0.9624$**

Repairman's busy period **$(B_0) = 0.2157$**

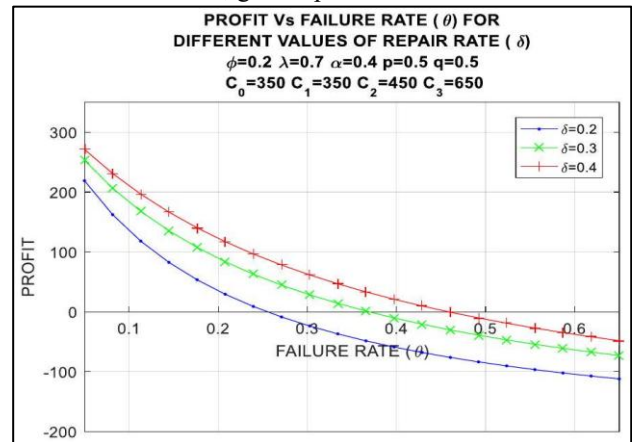
Repairman's Visits expected **$(V_0) = 0.0618$**

Replacements expected **$(RP_0) = 0.0077$**

Figure 2 shows variation in profit with failure rate (θ) when the repair rate (δ) values are fixed. Decrease in failure rate increases profit, also more profit, for high values of (δ).

Conclusions obtained

- For $\delta = 0.2, P > \text{or} = \text{or} < 0$ as $\theta < \text{or} = \text{or} > 0.256$. Profitable system if $\theta < 0.256$.
- For $\delta = 0.3, P > \text{or} = \text{or} < 0$ as $\theta < \text{or} = \text{or} > 0.370$. Hence, the system is profitable if $\theta < 0.370$.
- For $\delta = 0.4, P > \text{or} = \text{or} < 0$ as $\theta < \text{or} = \text{or} > 0.459$. Thus, gain is possible if $\theta < 0.459$.

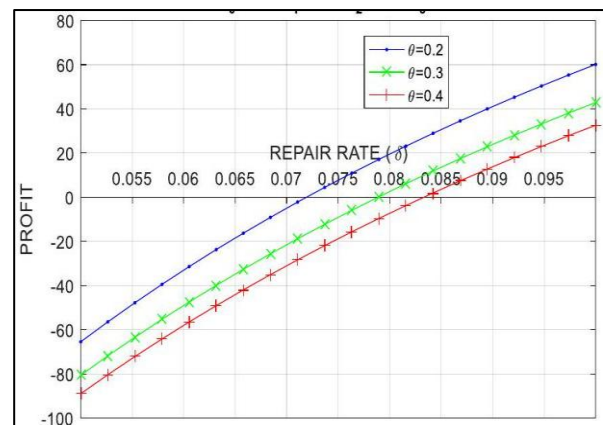


[Fig.2: Profit vs Failure Rate]

Figure 3 is variation of profit by varying repair rate (δ) when the failure rate is fixed (θ). Decrease in repair rate (δ) leads to a decrement in profit. Profit is higher if the failure rate (θ) is low.

Some observations are:

- For $\theta = 0.2, P < \text{or} = \text{or} > 0$ as $\delta < \text{or} = \text{or} > 0.072$. The system is profitable if $\delta > 0.072$.
- For $\theta = 0.3, P < \text{or} = \text{or} > 0$ as $\delta < \text{or} = \text{or} > 0.079$. The system is beneficial if $\delta > 0.079$.
- For $\theta = 0.4, P < \text{or} = \text{or} > 0$ as $\delta < \text{or} = \text{or} > 0.083$. The model is profitable if $\delta > 0.083$.

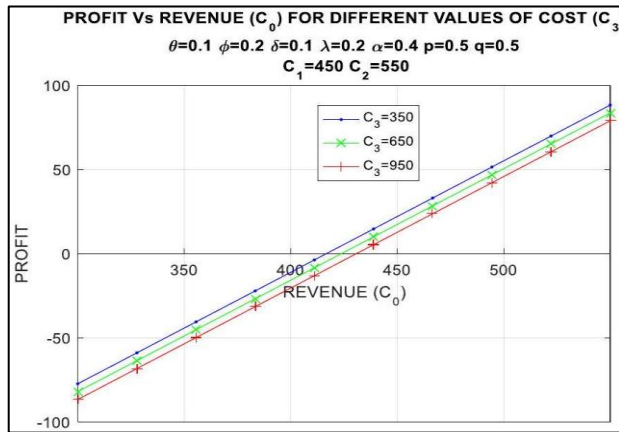


[Fig.3 Profit vs Repair Rate]

Figure 4 depicts profit P vs. revenue per uptime unit (C_0) and fixed cost is (C_3). Decrease in revenue uptime (C_0) decreases P . It is higher for lower values of (C_3).

So, results obtained

- (i) For $C_3 = 350$, profit P is $>$ or $=$ or < 0 as C_0 is $>$ or $=$ or < 416.798 . For gain $C_0 > 416.798$.
- (ii) For $C_3 = 650$, profit P is $>$ or $=$ or < 0 as C_0 is $>$ or $=$ or < 423.855 . The company is in profit if $C_0 > 423.855$.
- (iii) For $C_3 = 950$, the profit P is $>$ or $=$ or < 0 as C_0 is $>$ or $=$ or < 430.916 . Hence $C_0 > 430.916$ for company benefit.



[Fig.4: Profit vs Revenue]

VII. CONCLUSION

Numerical values and graphical analysis show that more system failures lead to higher costs, lower productivity, and ultimately reduced profit. Investing in a higher repair rate or maintenance can prevent failures and downtime. More revenue leads to higher profits. Repairing and replacing substandard or faulty components can improve overall efficiency. When the system runs smoothly, it yields better results. Cut-off points likely refer to specific decision-making criteria used to achieve gains.

DECLARATION STATEMENT

Some of the cited references are older and are noted explicitly as [1], [2], [4], [5], [6], [7], [8], [10], [11] and [12]. However, these works remain significant for the current study, as they are pioneering in their fields.

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- **Data Access Statement and Material Availability:** The adequate resources of this article are publicly accessible.
- **Author's Contributions:** The authorship of this article is contributed solely.

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