



Ramanujan's Ellipse Formula and Radical Bounds

K. Srinivasa Raghava, R. Sivaraman*

Abstract: Let $P(a, b)$ be the perimeter of an ellipse with semiaxes $a \geq b > 0$, and set $h = \left(\frac{a-b}{a+b}\right)^2$. The normalized perimeter is the hypergeometric function $F(h) = \frac{P(a,b)}{\pi(a+b)} = {}_2F_1\left(-\frac{1}{2}, -\frac{1}{2}; 1; h\right)$. We first characterize Ramanujan's radical $3 - \sqrt{4-h}$ as the largest global lower bound in a natural family of tangent-matched one-radical functions. We then introduce the quadratic-radical function $S(h) = 2 - \sqrt{1 - \frac{h}{2} + \frac{h^2}{32}}$. It is the unique radical with a quadratic radicand that agrees with F through h^3 . A coefficient argument proves $3 - \sqrt{4-h} < S(h) < F(h)$ ($0 < h < 1$) and gives the best possible estimate $\frac{7}{16384}h^4 < F(h) - S(h) < \left(\frac{4}{\pi} - 2 + \sqrt{\frac{17}{32}}\right)h^4$. Thus the new lower approximation has an eccentricity error of order e^{16} , compared with order e^{12} for Ramanujan's first formula. Finally, we give a canonical construction of higher-degree radical approximants and prove their local existence and uniqueness.

Keywords: Ellipse Perimeter, Ramanujan Approximation, Hypergeometric Function, Algebraic Approximation, Sharp Inequality.

I. INTRODUCTION

The perimeter of an ellipse is a classic example in which a simple geometric question leads to a special function. If $a \geq b > 0$ are the semiaxes and

$$e = \sqrt{1 - \frac{b^2}{a^2}},$$

then $P(a, b) = 4a \int_0^{\pi/2} \sqrt{1 - e^2 \sin^2 t} dt$. (1)

The integral is a complete elliptic integral of the second kind. Its analytic and numerical theory is well developed; see Carlson [5, account in §19.36], Borwein and Borwein [2, account in § 2], Adlaj [1], and the NIST handbook [5, current online companion, Ch. 19].

Elementary approximations to (1) have been studied for more than two centuries. Ivory found a rapidly convergent series [7, historical account and proof in § 2.2], and Muir proposed an influential algebraic approximation [4, historical discussion in § 1.1]. Historical accounts and broad comparisons are given by Almkvist and Berndt [5, historical account in § 19.9(i)],

Berndt [3, historical source cited in § 1.1], and Sýkora [4, survey and comparison in § 1.1]. Sharp inequalities were developed by Barnard, Pearce, and Richards [5, historical account in § 19.9(i)], Barnard, Pearce, and Schovanec [5, original work cited in § 19.9(i)], Villarino [7, historical account in § 1], and Jameson [6, discussion in § 2.1.2]. Recent work continues to seek a useful balance between simplicity and accuracy [4, 7, 8, 9]. Ramanujan's first approximation is

$$P(a, b) \approx \pi \left(3(a+b) - \sqrt{(3a+b)(a+3b)} \right). \quad (2)$$

It appeared in his 1914 paper [4, historical account in § 1.2]. Ramanujan described his ellipse formulas as empirical and did not derive them. Continued fractions give a natural reconstruction; related explanations may be found in Villarino [4, discussion in § 3.1] and Shankar [7, §§ 3–4], with general background in Wall [5, source cited in 3.10(iii)] and Cuyt et al. [5, source cited in § 3.10(ii)].

The right variable for the present problem is

$$h = \left(\frac{a-b}{a+b} \right)^2, \quad 0 \leq h < 1. \quad (3)$$

After division by $\pi(a+b)$, the exact perimeter and Ramanujan's first expression become

$$F(h) = {}_2F_1\left(-\frac{1}{2}, -\frac{1}{2}; 1; h\right), R(h) = 3 - \sqrt{4-h}. \quad (4)$$

It is known that $R(h) < F(h)$ on $0 < h < 1$, and the best constants in the corresponding error estimate follow from the general work of Barnard, Pearce, and Schovanec [5]. We include a short coefficient proof because it is the starting point for the stronger approximation below.

The main contribution of this paper is the quadratic-radical function:

$$S(h) = 2 - \sqrt{1 - \frac{h}{2} + \frac{h^2}{32}}. \quad (5)$$

It is forced by one additional order of coefficient matching. We prove the strict chain:

$$R(h) < S(h) < F(h), \quad 0 < h < 1, \quad (6)$$

and the sharp bound

$$\frac{7}{16384}h^4 < F(h) - S(h) < \left(\frac{4}{\pi} - 2 + \sqrt{\frac{17}{32}} \right)h^4. \quad (7)$$

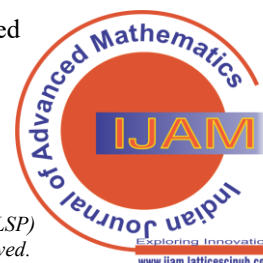
We also show that R is extremal among all one-radical lower bounds that have the correct value and derivative at the circle. The final section places R and S in a canonical sequence of higher-degree radical approximants. For the general sequence we assert only the local matching property; global positivity is proved here for the first two members.

II. THE EXACT NORMALIZED PERIMETER

We begin with the exact series in the variable h .

Proposition 1.

For $a \geq b > 0$, with h defined by (3),



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$$F(h) = \frac{P(a,b)}{\pi(a+b)} = \sum_{n=0}^{\infty} \binom{1/2}{n}^2 h^n = {}_2F_1\left(-\frac{1}{2}, -\frac{1}{2}; 1; h\right), \quad (8)$$

In particular,

$$F(h) = 1 + \frac{h}{4} + \frac{h^2}{64} + \frac{h^3}{256} + \frac{25h^4}{16384} + \frac{49h^5}{65536} + \dots, \quad (9)$$

and

$$\lim_{h \rightarrow 1^-} F(h) = \frac{4}{\pi} \quad (10)$$

Proof. Set

$$u = \frac{a-b}{a+b}, \quad h = u^2.$$

The standard parametrization of the ellipse gives

$$P(a, b) = \int_0^{2\pi} \sqrt{a^2 \sin^2 t + b^2 \cos^2 t} \, dt.$$

A direct calculation yields

$$a^2 \sin^2 t + b^2 \cos^2 t = \frac{(a+b)^2}{4} (1 - ue^{2it})(1 - ue^{-2it}).$$

Since $0 \leq u < 1$, the two binomial series converge uniformly in t . Termwise integration is therefore valid, and

$$\begin{aligned} P(a, b) &= \frac{a+b}{2} \sum_{m,n \geq 0} (-1)^{m+n} \binom{1/2}{m} \binom{1/2}{n} u^{m+n} \int_0^{2\pi} e^{2i(m-n)t} \, dt \\ &= \pi(a+b) \sum_{n=0}^{\infty} \binom{1/2}{n}^2 u^{2n} \end{aligned}$$

This proves the series. The hypergeometric form follows from the definition of ${}_2F_1$, and expansion gives (9).

Gauss's summation formula gives

$${}_2F_1\left(-\frac{1}{2}, -\frac{1}{2}; 1; 1\right) = \frac{\Gamma(1)\Gamma(2)}{\Gamma(3/2)^2} = \frac{4}{\pi},$$

which proves (10).

III. RAMANUJAN'S RADICAL

We record the coefficient comparison for R . Besides recovering the known lower bound, it will be used in the extremal characterization below.

Lemma 2.

Write $F(h) = \sum_{n=0}^{\infty} c_n h^n$, $R(h) = \sum_{n=0}^{\infty} r_n h^n$. Then $c_n = r_n$ for $n = 0, 1, 2$, and $c_n > r_n > 0$ ($n \geq 3$).

Proof. Proposition 1 gives

$$c_n = \binom{1/2}{n}^2.$$

Since

$$R(h) - 1 = 2 \left(1 - \sqrt{1 - \frac{h}{4}} \right),$$

the Catalan generating function gives, for $n \geq 1$,

$$r_n = \frac{1}{n 2^{4n-2}} \binom{2n-2}{n-1}. \quad (11)$$

The first two positive coefficients agree. Direct calculation gives

$$\frac{c_{n+1}}{c_n} = \frac{(2n-1)^2}{4(n+1)^2}, \quad \frac{r_{n+1}}{r_n} = \frac{2n-1}{8(n+1)}.$$

Consequently,

$$\frac{c_{n+1}/r_{n+1}}{c_n/r_n} = \frac{2(2n-1)}{n+1}.$$

This ratio equals 1 at $n = 1$ and is greater than 1 for $n \geq 2$. Since $c_2 = r_2$, the result follows.

Proposition 3.

For $0 < h < 1$,

$$\frac{h^3}{512} < F(h) - R(h) < \left(\frac{4}{\pi} - 3 + \sqrt{3} \right) h^3. \quad (12)$$

Both constants are best possible.

Proof. Lemma 2 shows that

$$\frac{F(h) - R(h)}{h^3} = (c_3 - r_3) + \sum_{n=4}^{\infty} (c_n - r_n) h^{n-3}$$

is strictly increasing on $0 < h < 1$. Its left endpoint limit is

$$c_3 - r_3 = \frac{1}{256} - \frac{1}{512} = \frac{1}{512}.$$

At the right endpoint, (10) gives

$$F(1) - R(1) = \frac{4}{\pi} - 3 + \sqrt{3}.$$

The endpoint limits prove both the inequalities and the sharpness.

The next result gives a global optimality statement for Ramanujan's radical.

Theorem 4.

Let $T(h) = \alpha - \sqrt{\beta - \gamma h}$ be real on $0 \leq h \leq 1$, with $\beta > 0$, and suppose $T(0) = 1, T'(0) = \frac{1}{4}$. Then $T(h) \leq F(h)$ for

every $0 \leq h < 1$ if and only if $T(h) = 1 + s - s\sqrt{1 - \frac{h}{2s}}$ for some $s \geq 2$. Among all these lower bounds, $R = T_2$ is pointwise largest.

Proof. Put $s = \sqrt{\beta}$. The two normalization conditions give

$$\alpha = 1 + s, \quad \gamma = \frac{s}{2},$$

so, every such function has the stated form. Its expansion begins

$$T(h) = 1 + \frac{h}{4} + \frac{h^2}{32s} + O(h^3). \quad (13)$$

If $T(h) \leq F(h)$ near $h = 0$, comparison with (9) forces

$$\frac{1}{32s} \leq \frac{1}{64},$$

and hence $s \geq 2$.

Conversely, let $s \geq 2$. If

$$1 - \sqrt{1-x} = \sum_{n=1}^{\infty} u_n x^n, \quad u_n > 0,$$

then

$$T(h) = 1 + \sum_{n=1}^{\infty} \frac{u_n}{2^n s^{n-1}} h^n.$$

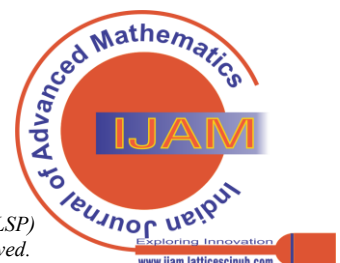
For $n \geq 2$, these coefficients decrease as s increases. Therefore

$$T(h) \leq T_2(h) = 3 - \sqrt{4-h} = R(h)$$

on $0 \leq h < 1$. Lemma 2 gives $R(h) \leq F(h)$. Thus every $s \geq 2$ gives a lower bound, and $s = 2$ gives the largest one.

Corollary 5.

The function $R(h) = 3 - \sqrt{4-h}$ is the unique





function of the form $\alpha - \sqrt{\beta - \gamma h}$ that agrees with $F(h)$ through h^2 .

Proof. The value and first derivative force the family in Theorem 4. Equality of the coefficient of h^2 in (13) forces $s = 2$.

IV. A QUADRATIC-RADICAL LOWER BOUND

We now pass from a linear to a quadratic radicand. This additional term raises the local order of accuracy and yields a stronger global bound.

Theorem 6.

Let

$$A(h) = C - \sqrt{q_0 + q_1 h + q_2 h^2}, \quad q_0 > 0,$$

where the positive square-root branch is used near $h = 0$. If $F(h) - A(h) = O(h^4)$ ($h \rightarrow 0$), then

$$C = 2, \quad q_0 = 1, \quad q_1 = -\frac{1}{2}, \quad q_2 = \frac{1}{32}.$$

Thus S in (5) is the unique member of this family that agrees with F through h^3 .

Proof. Let $Q(h) = q_0 + q_1 h + q_2 h^2$. At $h = 0$,

$$\sqrt{q_0} = C - 1 > 0.$$

The matching assumption gives

$$\sqrt{Q(h)} = C - F(h) + O(h^4).$$

After squaring,

$$Q(h) = (C - F(h))^2 + O(h^4). \quad (14)$$

Since Q has degree at most 2, the coefficient of h^3 on the right must vanish. Using (9), that coefficient is

$$-\frac{C-1}{128} + \frac{1}{128}.$$

Hence $C = 2$. The coefficients through h^2 in $(2 - F(h))^2$ are

$$1 - \frac{h}{2} + \frac{h^2}{32}.$$

Equation (14) now forces the stated values of q_0, q_1, q_2 .

The central coefficient comparison requires only the binomial series and an elementary geometric estimate.

Lemma 7.

Write

$$S(h) = \sum_{n=0}^{\infty} s_n h^n.$$

Then $s_n = c_n$ for $n = 0, 1, 2, 3$, and

$$s_n < c_n \quad (n \geq 4).$$

Proof. For $n \geq 1$, put

$$u_n = \left| \binom{1/2}{n} \right|.$$

Then $c_n = u_n^2$, and

$$\sqrt{1-x} = 1 - \sum_{n=1}^{\infty} u_n x^n.$$

Factor the radicand of S as

$$1 - \frac{h}{2} + \frac{h^2}{32} = (1 - \alpha h)(1 - \beta h)$$

$$\alpha = \frac{2 + \sqrt{2}}{8}, \quad \beta = \frac{2 - \sqrt{2}}{8}.$$

Multiplication of the two binomial series gives, for $n \geq 1$,

$$s_n = u_n(\alpha^n + \beta^n) - \sum_{k=1}^{n-1} u_k u_{n-k} \alpha^k \beta^{n-k}. \quad (15)$$

In particular,

$$s_n < u_n(\alpha^n + \beta^n) \leq 2u_n \alpha^n \quad (n \geq 2).$$

Direct expansion gives

$$s_4 = \frac{9}{8192} < \frac{25}{16384} = c_4, \quad s_5 = \frac{11}{32768} < \frac{49}{65536} = c_5.$$

For the remaining coefficients, note that

$$u_6 = \frac{21}{1024}, \quad u_6 - 2\alpha^6 = \frac{237 - 70\sqrt{2}}{16384} > 0.$$

Moreover,

$$\frac{u_{n+1}}{u_n} = \frac{2n-1}{2(n+1)} > \alpha \quad (n \geq 6).$$

Induction therefore gives $u_n > 2\alpha^n$ for every $n \geq 6$.

Combining this with (15) yields

$$s_n < 2u_n \alpha^n < u_n^2 = c_n$$

for all $n \geq 6$. The equality of the first four coefficients follows from Theorem 6.

Theorem 8.

For $0 < h < 1$,

$$R(h) < S(h) < F(h), \quad (16)$$

and

$$\frac{7}{16384} h^4 < F(h) - S(h) < \left(\frac{4}{\pi} - 2 + \sqrt{\frac{17}{32}} \right) h^4. \quad (17)$$

Both constants in (17) are best possible.

Proof. Lemma 7 gives

$$F(h) - S(h) = \sum_{n=4}^{\infty} (c_n - s_n) h^n > 0.$$

It also shows that

$$\frac{F(h) - S(h)}{h^4} = (c_4 - s_4) + \sum_{n=5}^{\infty} (c_n - s_n) h^{n-4}$$

is strictly increasing on $0 < h < 1$. Its left endpoint limit is

$$c_4 - s_4 = \frac{25}{16384} - \frac{18}{16384} = \frac{7}{16384}.$$

At the right endpoint,

$$F(1) - S(1) = \frac{4}{\pi} - 2 + \sqrt{\frac{17}{32}}.$$

This proves (17) and the sharpness of both constants.

It remains to compare S with R . Put

$$B(h) = 4 - \frac{h}{2} - \frac{h^2}{32}.$$

For $0 < h < 1$, $B(h) > 0$, and

$$B(h)^2 - 4(4-h) = \frac{h^3(h+32)}{1024} > 0.$$

Thus $B(h) > 2\sqrt{4-h}$. Equivalently,

$$(\sqrt{4-h} - 1)^2 > 1 - \frac{h}{2} + \frac{h^2}{32}.$$

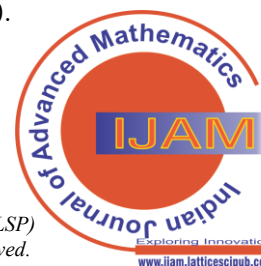
Both sides being nonnegative, we obtain

$$\sqrt{4-h} - 1 > \sqrt{1 - \frac{h}{2} + \frac{h^2}{32}},$$

which is exactly $R(h) < S(h)$.

Corollary 9.

Let $a > b > 0$, and define h by (3). Then,



$$\pi(a+b) \left(S(h) + \frac{7}{16384} h^4 \right) < P(a,b)$$

$$< \pi(a+b) \left(S(h) + \left(\frac{4}{\pi} - 2 + \sqrt{\frac{17}{32}} \right) h^4 \right) \quad (18)$$

The constants are best possible uniformly over $0 < h < 1$. In particular, $\pi(a+b)R(h) < \pi(a+b)S(h) < P(a,b)$.

Proof. Multiply Theorem 8 by $\pi(a+b)$.

Corollary 10.

As $e \rightarrow 0$,

$$P(a,b) - \pi(a+b)S(h) = \frac{7\pi a}{2^{29}} e^{16} + O(ae^{18}). \quad (19)$$

Proof. Since $b/a = \sqrt{1-e^2}$,

$$h = \left(\frac{1 - \sqrt{1-e^2}}{1 + \sqrt{1-e^2}} \right)^2 = \frac{e^4}{16} + O(e^6),$$

and

$$a+b = 2a + O(ae^2).$$

Lemma 7 gives

$$F(h) - S(h) = \frac{7}{16384} h^4 + O(h^5).$$

Therefore

$$\begin{aligned} & \pi(a+b)(F(h) - S(h)) \\ &= \pi(2a + O(ae^2)) \left(\frac{7}{2^{14}} \frac{e^{16}}{2^{16}} + O(e^{18}) \right) \end{aligned}$$

which simplifies to (19).

Remark 11. The leading eccentricity error for Ramanujan's first formula is

$$P(a,b) - \pi(a+b)R(h) = \frac{\pi a}{2^{20}} e^{12} + O(ae^{14}).$$

Thus, the quadratic radicand raises the local order from e^{12} to e^{16} . The result is structural rather than a numerical fit: every coefficient in the defining radical is forced by exact matching.

V. A CANONICAL RADICAL SEQUENCE

The preceding two radicals are the first members of a general local construction. This section proves existence and uniqueness without asserting a global inequality for the higher members. For a power series $G(h)$, let $[G]_{\leq m}$ denote its Taylor polynomial of degree m at the origin. Retain the notation

$$F(h) = \sum_{n=0}^{\infty} c_n h^n, \quad c_n = \binom{1/2}{n}.$$

Theorem 12.

For every integer $m \geq 1$, define

$$C_m = \frac{\sum_{j=0}^{m+1} c_j c_{m+1-j}}{2c_{m+1}} \quad (20)$$

and

$$Q_m(h) = [(C_m - F(h))^2]_{\leq m}. \quad (21)$$

then $C_m > 1$, and the function

$$A_m(h) = C_m - \sqrt{Q_m(h)}, \quad (22)$$

with $\sqrt{Q_m(0)} = C_m - 1$, is analytic near $h = 0$ and satisfies

$$F(h) - A_m(h) = O(h^{m+2}). \quad (23)$$

Moreover, A_m is the unique function of the form

$$C - \sqrt{Q(h)}, \quad C > 1, \quad \deg Q \leq m,$$

that satisfies (23).

Proof. All coefficients c_n are positive. The two endpoint terms in the numerator of (20) sum to $2c_{m+1}$, and the remaining terms are positive. Hence $C_m > 1$.

For $n \geq 1$,

$$[h^n](C - F(h))^2 = -2Cc_n + \sum_{j=0}^n c_j c_{n-j}.$$

The choice (20) therefore gives

$$[h^{m+1}](C_m - F(h))^2 = 0. \quad (24)$$

By (21) and (24),

$$Q_m(h) = (C_m - F(h))^2 + O(h^{m+2}).$$

Since $C_m - F(0) = C_m - 1 > 0$,

taking the analytic square root gives

$$\sqrt{Q_m(h)} = C_m - F(h) + O(h^{m+2}),$$

which proves (23).

For uniqueness, suppose

$$A(h) = C - \sqrt{Q(h)}$$

has $\deg Q \leq m$, $C > 1$, and $F - A = O(h^{m+2})$. Then

$$Q(h) = (C - F(h))^2 + O(h^{m+2}).$$

Because Q has no term in h^{m+1} , the coefficient of that term on the right must vanish. The displayed coefficient formula forces $C = C_m$. Matching the coefficients of degrees $0, \dots, m$ then forces $Q = Q_m$.

Proposition 13.

With the notation of Theorem 12,

$$F(h) - A_m(h) = \lambda_m h^{m+2} + O(h^{m+3}),$$

where

$$\lambda_m = -\frac{[h^{m+2}](C_m - F(h))^2}{2(C_m - 1)}. \quad (25)$$

In particular,

$$A_1(h) = R(h), \quad \lambda_1 = \frac{1}{512},$$

and

$$A_2(h) = S(h), \quad \lambda_2 = \frac{7}{16384}.$$

Proof. Write

$$(C_m - F(h))^2 = Q_m(h) + d_m h^{m+2} + O(h^{m+3}),$$

where

$$d_m = [h^{m+2}](C_m - F(h))^2.$$

Since

$$C_m - F(0) = C_m - 1,$$

$$\sqrt{Q_m(h)} = C_m - F(h) - \frac{d_m}{2(C_m - 1)} h^{m+2} + O(h^{m+3}).$$

Substitution into (22) proves (25). The cases $m = 1, 2$ follow by direct use of (20) and (21).

Corollary 14.

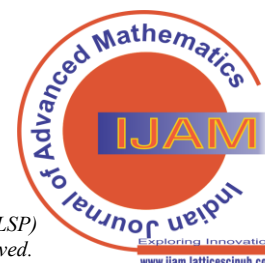
For each fixed $m \geq 1$, the local ellipse-perimeter error produced by A_m is

$$P(a,b) - \pi(a+b)A_m(h) = O(ae^{4m+8}) \quad (e \rightarrow 0).$$

Proof. Theorem 12 gives an error $O(h^{m+2})$, while

$$h = \frac{e^4}{16} + O(e^6),$$

$$a+b = 2a + O(ae^2).$$





Remark 15.

Theorem 12 is a local algebraic construction. For $m \geq 3$, positivity of Q_m on the full interval, the sign of $F - A_m$, and convergence of the sequence are separate questions. No such global claim is needed for Theorems 4 and 8.

VI. CONCLUSION

Ramanujan's first ellipse formula is not merely a successful numerical expression. It is the largest lower bound in the complete one-radical family that matches the circle's value and tangent. Passing to a quadratic radicand gives the canonical refinement

$$2 - \sqrt{1 - \frac{h}{2} + \frac{h^2}{32}},$$

which lies strictly between Ramanujan's first radical and the exact normalized perimeter. Its coefficient error is positive from the first omitted term onward, producing a sharp global enclosure and an e^{16} local error.

The higher-degree construction shows that both radicals arise from one algebraic principle: choose the constant so that the first unavailable coefficient in the squared relation vanishes. This provides a systematic sequence of locally optimal radical approximants while keeping the global assertions precise. The first two members are completely controlled; the higher members naturally raise further questions about positivity, ordering, and uniform convergence.

DECLARATION STATEMENT

After aggregating input from all authors, I must verify the accuracy of the following information as the article's author.

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- **Ethical Approval and Consent to Participate:** The data provided in this article is exempt from the requirement for ethical approval or participant consent.
- **Data Access Statement and Material Availability:** The data and materials in this article are publicly accessible.
- **Authors' Contributions:** The authorship of this article is contributed equally to all participating individuals.

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