

# On Group $A$ -Cordial Labelling of Double Uniform $(t_1^{l_1}, t_2^{l_2})$ - Ply

Samina Boxwala, Aditi S. Phadke, Pramod N. Shinde, Nilesh Mundlik

**Abstract:** Let  $A$  denote the multiplicative group  $\{1, -1, i, -i\}$ . In this paper, we define the notion of double uniform  $(t_1^{l_1}, t_2^{l_2})$ -ply and prove that it is a group  $A$ -cordial with each path of length at least 5 by explicitly giving labellings for all the possible cases that arise. *Mathematics Subject Classification [2020]: Primary 05C78*

**Keywords:**  $t$ -ply; Uniform  $t$ -ply; Double  $t$ -ply; Double Uniform  $t$ -ply; Group  $A$ -Cordial Labelling.

## I. INTRODUCTION

Cahit introduced cordial labellings [5] in 1987 as a weakened version of graceful labellings. Different families of cordial labellings and its variations were discussed in [1] [2] and [3].

**Definition 1.1.** Let  $f: V(G) \rightarrow \{0,1\}$  be any function. To each edge  $ab$  assign the label  $|f(a) - f(b)|$ . Let  $v_f(0), v_f(1)$  denote the number of vertices in  $G$  with the labels 0 and 1, respectively. Let  $e_f(0), e_f(1)$  denote the number of edges in  $G$  with the labels 0 and 1, respectively. The function  $f$  is called a cordial labelling if  $|v_f(0) - v_f(1)| \leq 1$  and  $|e_f(0) - e_f(1)| \leq 1$ . The graph  $G$  is called a cordial graph if it admits a cordial labelling. (See [5]).

Prime labellings were first introduced by Roger Entringer and discussed in a paper by Tout et al. [1].

**Definition 1.2.** Let  $G = (V, E)$  be a graph. A bijection  $f: V \rightarrow \{1, 2, \dots, |V|\}$  is called a prime labelling if for each  $e = \{u, v\} \in E$ , we have  $\gcd(f(u), f(v)) = 1$ . A graph that admits a prime labelling is called a prime graph.

Motivated by these two labellings, the notion of a group  $A$ -Cordial labelling of graphs was introduced by M. K. Karthik Chidambaram, S. Athisayanathan and R. Ponraj (See [8] [9]).

**Definition 1.3.** Let  $G$  be a graph and let  $A$  be a group. Let  $o(a)$  denote the order of an element  $a \in A$ . Let  $f: V(G) \rightarrow A$  be a function. For each edge  $uv$ , assign the label 1 if  $\gcd(o(f(u)), o(f(v))) = 1$  and 0 otherwise. Let  $v_f(a)$  denote the number of vertices in  $G$  labeled by  $f$  with the element  $a$  of the group  $A$ . Let  $e_f(0), e_f(1)$  denote the number of edges with labels 0 and 1, respectively. The function  $f$  is called a group  $A$ -cordial labelling if  $|v_f(a) - v_f(b)| \leq 1$  for all  $a, b \in A$  and  $|e_f(0) - e_f(1)| \leq 1$ . A graph which admits a group  $A$ -Cordial labelling is called group  $A$ -cordial.

Throughout this paper, the group under consideration will be  $A = \{1, -1, i, -i\}$  With respect to multiplying complex numbers.

For the multiplicative group  $A = \{1, -1, i, -i\}$  the group  $A$ -Cordial labelling for different graphs was considered by Karthik et. al. ([8] [9]) and Boxwala et. al. [4] have obtained a group  $A$ -cordial labelling for the uniform  $t$ -ply. Further, for the group  $A = S_3$ , group  $A$ -Cordial labelling was studied by B Chandra and R Kala studied cordial labelling ([6], [7]).

**Remark 1.4.** Note that for the group  $A = \{1, -1, i, -i\}$  with respect to the multiplication of complex numbers  $o(1) = 1, o(-1) = 2$  and  $o(i) = o(-i) = 4$ . Thus, an edge  $uv$  will be assigned label 1 if at least one of the vertices  $u$  or  $v$  has been labelled 1. Hence, permuting the labels.  $-1, i, -i$  the choice of the vertices among themselves will not affect the edge labelling.

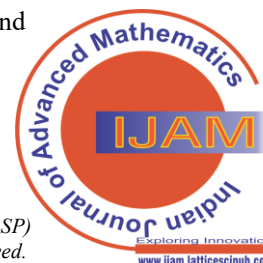
## II. PRELIMINARIES

**Definition 2.1.** (See [1]) A  $t$ -ply  $P_t(u, w)$  is a graph with  $t$  paths, each of length at least two, such that no two paths have a vertex in common except for the end vertices  $u$  and  $w$  (common to all), which will also be referred to as the source and sink vertices, respectively. We note in passing that.  $P_t(u, w)$  denotes an entire family of  $t$ -ply graphs with  $t$  paths and end vertices  $u, w$ .

**Definition 2.2.** A path on the  $t$ -ply graph  $P_t(u, w)$  is said to be of type  $l$  if the length of the path is congruent to  $l$  modulo 4,  $l = 1, 2, 3, 4$ . We call  $P_t(u, w)$  a uniform  $t$ -ply of type  $l$  if all  $t$  paths are of type  $l$  i.e., the length of each path is congruent to  $l$  modulo 4. We denote such a  $t$ -ply by  $P_t^l(u, w)$ .

**Remark 2.3.** We observe that in a uniform  $t$ -ply of type  $l$ , the  $x^{\text{th}}$  path is of length  $n_x + 1$  where  $n_x + 1 \equiv l \pmod{4}$ .

$$\text{Let } n_x + 1 = 4k_x + l; \text{ then } |V(P_t^l(u, w))| = 2 + 4 \sum_{x=1}^t k_x + (l-1)t \text{ and } |E(P_t^l(u, w))| = 4 \sum_{x=1}^t k_x + lt.$$



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## On Group A-Cordial Labeling of Double Uniform $(t_1^{l_1}, t_2^{l_2}) - \text{Ply}$

Thus, the number of edges will be odd if and only if both  $l$  and  $t$  are odd.

**Theorem 2.4.** Every uniform  $t$ -ply with each path of length at least 5 is group A-cordial. (See [4]).

Theorem 2.4 has been proved in [4] by giving an explicit labelling for each of the 12 cases that arose. These 12 labellings are listed in Table I, as they will be useful for proving the main result of this paper. We also remark that, in this table, we state the conditions for cordiality satisfied by the vertex and edge labels for each function in terms of parity. In each row, we choose  $r$  such that  $t \equiv r \pmod{2}$  or  $t \equiv r \pmod{4}$  as the case may be. The relation between the corresponding vertex and edge labels for each label is shown

**Table I: Library of Labellings for Uniform  $t$ -ply**

$l$	$t$	$f$	$v_f(-1)$	$v_f(1)$	$v_f(t)$	$v_f(-t)$	Relation between $e_f(0)$ and $e_f(1)$
1	$t \equiv 0 \pmod{2}$	$\alpha_{10}$	1	1	0	0	$e_{\alpha_{10}}(0) = e_{\alpha_{10}}(1)$
	$t \equiv 1 \pmod{2}$	$\alpha_{11}$	1	1	0	0	$e_{\alpha_{11}}(0) + 1 = e_{\alpha_{11}}(1)$
2	$t \equiv 0 \pmod{4}$	$\alpha_{20}$	1	1	0	0	$e_{\alpha_{20}}(0) = e_{\alpha_{20}}(1)$
	$t \equiv 1 \pmod{4}$	$\alpha_{21}$	1	1	1	0	$e_{\alpha_{21}}(0) = e_{\alpha_{21}}(1)$
	$t \equiv 2 \pmod{4}$	$\alpha_{22}$	0	0	0	0	$e_{\alpha_{22}}(0) = e_{\alpha_{22}}(1)$
	$t \equiv 3 \pmod{4}$	$\alpha_{23}$	0	1	0	0	$e_{\alpha_{23}}(0) = e_{\alpha_{23}}(1)$
3	$t \equiv 0 \pmod{2}$	$\alpha_{30}$	1	1	0	0	$e_{\alpha_{30}}(0) = e_{\alpha_{30}}(1)$
	$t \equiv 1 \pmod{2}$	$\alpha_{31}$	0	0	0	0	$e_{\alpha_{31}}(0) = e_{\alpha_{31}}(1) + 1$
4	$t \equiv 0 \pmod{4}$	$\alpha_{40}$	1	1	0	0	$e_{\alpha_{40}}(0) = e_{\alpha_{40}}(1)$
	$t \equiv 1 \pmod{4}$	$\alpha_{41}$	0	1	0	0	$e_{\alpha_{41}}(0) = e_{\alpha_{41}}(1)$
	$t \equiv 2 \pmod{4}$	$\alpha_{42}$	0	0	0	0	$e_{\alpha_{42}}(0) = e_{\alpha_{42}}(1)$
	$t \equiv 3 \pmod{4}$	$\alpha_{43}$	1	1	1	0	$e_{\alpha_{43}}(0) = e_{\alpha_{43}}(1)$

We begin by revisiting the definitions of two functions  $g$  and  $h$  which featured prominently in the various stages.

For any path  $P = \{v_1, v_2, \dots, v_n\}$  of length  $n - 1$ ; we take  $n = 4k + m$ , where  $m = 0, 1, 2, 3$ .

Let  $g: V(P) \rightarrow A = \{1, -1, i, -i\}$  be a map defined as follows:

For  $1 \leq j \leq 4k$ ,

$$\begin{aligned} g(v_j) &= i, \text{ if } j \equiv 0 \pmod{4} \\ &= 1, \text{ if } j \equiv 1 \pmod{4} \\ &= -1, \text{ if } j \equiv 2 \pmod{4} \\ &= -i, \text{ if } j \equiv 3 \pmod{4}. \end{aligned}$$

Likewise, we have

$$\begin{aligned} h(v_j) &= i, \text{ if } j \equiv 0 \pmod{4} \\ &= -1, \text{ if } j \equiv 1 \pmod{4} \\ &= -i, \text{ if } j \equiv 2 \pmod{4} \\ &= 1, \text{ if } j \equiv 3 \pmod{4}. \end{aligned}$$

Observe that  $g$  and  $h$  label only the first  $4k$  vertices on the path.

Consider the uniform  $t$ -ply  $P_t^l(u, w)$ . For  $l = 1, 3$ ; we have taken the number of paths  $t$  congruent modulo 2 i.e.  $t = 2s + r$ ; where  $r = 0, 1$ . We use the function  $g$  on the intermediate vertices  $v(x, 1), v(x, 2), \dots, v(x, 4k_x)$  of the first  $s$  paths i.e. for  $1 \leq x \leq s$  and the function  $h$  on the intermediate vertices of the paths from  $s + 1$  to  $2s$ . For  $l = 2, 4$ ; we have taken the number of paths  $t$  congruent modulo 4 ; i.e.  $t = 4s + r$ ; where  $r = 0, 1, 2, 3$ . We use the function  $g$  on the first  $2s$  paths i.e. for  $1 \leq x \leq 2s$  and the function  $h$  on the intermediate vertices of the next  $2s$  paths i.e.  $2s < x \leq 4s$ . This is stage I of the labelling. In this stage, the labels for the vertices and edges are distributed equitably.

alongside it. Also, we state the conditions for cordiality satisfied by the vertex and edge labels for each function in terms of parity.

### A. Additional Labellings Required

In addition to the labellings listed above, we will require two additional labellings in some cases that arise for the graph considered in this paper. These new labellings will be denoted by  $\alpha_{11}^*$  (Type A') and  $\alpha_{31}^*$  (Type C') and they are summarized in table II. For uniform  $t$ -ply the labelling was done in three stages. We now describe these labellings below, and since we will be using the same stages for the additional labellings, we mention the details of the stages below:

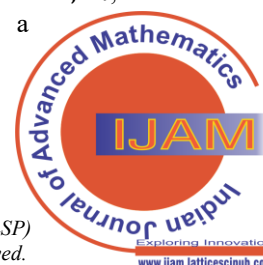
There now remain  $l - 1$  vertices for labelling on each of the paths that have received labelling in stage I. We label these  $2s(l - 1)$  or  $4s(l - 1)$  vertices, as the case may be, in stage II, which is discussed case-wise in [4]. In stage II of the labelling, we ensure that the vertex and edge labels are distributed equitably. We then look at the last  $r$  paths that have not yet been labelled. In stage III, we label all vertices on such paths. We label the vertex  $u$  with  $-1$  and the vertex  $w$  with  $1$  in all cases. In each case, the resulting labelling for  $P_t^l(u, w)$  was denoted by  $\alpha_{lr}$ .

We now come to the two specific cases where we require additional labelling as mentioned above. To distinguish the additional labellings from the ones given in [4], we use the superscript  $*$  for the same.

#### Case 1: $l = 1, r = 1$

We label the vertices in stages I and II as before. On the one remaining path, we use the function  $h$ . We use the label  $i$  for the source vertex instead of our usual  $-1$ . We get the labelling  $\alpha_{11}^*$ , for which  $v_{\alpha_{11}^*}(i) = v_{\alpha_{11}^*}(1) = v_{\alpha_{11}^*}(-1) + 1 = v_{\alpha_{11}^*}(-i) + 1$  and  $e_{\alpha_{11}^*}(0) = e_{\alpha_{11}^*}(1) + 1$

**Case 2:  $l = 3, r = 1$**  We first note that there will be at least 3 paths in this case. As in all other cases, we label the start equation  $u$  as minus 1 and  $u$  as  $-1$  and  $w$  as  $1$ . In stage I, we use the labelling  $g$  on the first  $s - 1$  paths followed by  $i$  and  $1$  on the last two vertices, and we use the labelling  $h$  on the next  $s - 1$  paths, followed by  $-1$  and  $-i$  on the last two vertices on each such path. On the three remaining paths, we use the label  $g$ . On one of these paths, we label the last two vertices as  $1, i$ , on the next with  $-1, -i$ , and on the last with  $i$  and  $-i$ . This gives us a labelling denoted by  $\alpha_{31}^*$  for which  $v_{\alpha_{31}^*}(-1) = v_{\alpha_{31}^*}(1) =$



$v_{\alpha_{31}}(i) = v_{\alpha_{31}}(-i)$  and  $e_{\alpha_{31}}(1) = e_{\alpha_{31}}(0) + 1$ .  
We describe them in Table II.

**Table II: Library of Additional Labellings for Uniform  $t$ -Ply**

Type	$t$	$f$	$v_f(-1)$	$v_f(1)$	$v_f(i)$	$v_f(-i)$	Relation between $e_f(0)$ and $e_f(1)$
A'	$t \equiv 1 \pmod{2}$	$\alpha_{11}^*$	0	1	1	0	$e_{\alpha_{11}^*}(0) = e_{\alpha_{11}^*}(1) + 1$
C'	$t \equiv 1 \pmod{2}$	$\alpha_{31}^*$	0	0	0	0	$e_{\alpha_{31}^*}(1) = e_{\alpha_{31}^*}(0) + 1$

**B. Group A-Cordiality of Double Uniform  $(t_1^{l_1}, t_2^{l_2})$  – Ply**

**Definition 3.1.** A double  $(t_1, t_2)$ -ply is a one-point union of two  $t$ -ply's  $P_{t_1}(u_1, w)$  and  $P_{t_2}(u_2, w)$  having only one vertex in common. We denote this double  $(t_1, t_2)$ -ply by  $P_{t_1}(u_1, w) \odot P_{t_2}(u_2, w)$ .

If  $P_{t_1}(u_1, w)$  is uniform of type  $l_1$  and  $P_{t_2}(u_2, w)$  is uniform of type  $l_2$  then the double  $(t_1, t_2)$ -ply is called a double uniform  $(t_1^{l_1}, t_2^{l_2})$ -ply and will be denoted by  $P_{t_1}^{l_1}(u_1, w) \odot P_{t_2}^{l_2}(u_2, w)$ .

**Theorem 3.2.** Every double uniform  $(t_1^{l_1}, t_2^{l_2})$ -ply with each path of length at least 5 is group A-cordial.

**Proof.** Consider a double uniform  $(t_1^{l_1}, t_2^{l_2})$  – ply  $P_{t_1}^{l_1}(u_1, w) \odot P_{t_2}^{l_2}(u_2, w)$ . To prove that this double uniform ply is a group A-cordial, we select an appropriate labelling from amongst the labellings mentioned in Table I for each ply separately. In each of these plys, we apply the labelling from the source to the sink vertex. This will ensure that in all cases  $w$  receives the label 1. Let  $F$  be the labelling that we choose for  $P_{t_1}^{l_1}(u_1, w)$  and  $G$  the labelling for  $P_{t_2}^{l_2}(u_2, w)$ . We then define the labelling for the vertices of  $P_{t_1}^{l_1}(u_1, w) \odot P_{t_2}^{l_2}(u_2, w)$  by

$$H(x) = F(x), \text{ if } x \in P_{t_1}^{l_1}(u_1, w)$$

$$= G(x), \text{ if } x \in P_{t_2}^{l_2}(u_2, w)$$

Note that in every labeling that features in Table I, the label for the vertex  $w$  is 1; hence  $H(w) = F(w) = G(w) = 1$ . As  $w$  is a shared vertex between the 2 plys,  $v_H(1) = v_F(1) + v_G(1) - 1$  and  $v_H(y) = v_F(y) + v_G(y)$  for  $y = -1, i, -i$ . Further, as there is no shared edge between the 2 plys,  $e_H(0) = e_F(0) + e_G(0)$ ,  $e_H(1) = e_F(1) + e_G(1)$ . We now break up the proof into two cases as given below:

**Case 1: At most one of the two ply’s has an Odd Number of edges**

In this case, the edge label balance will be achieved automatically by  $H$  if the edge label balance is achieved by  $F$  and  $G$  individually. For this case, we will restrict the choice of  $F$  and  $G$  to the labellings given in Table I. We can now focus only on achieving the vertex label balance in the double uniform ply by appropriately choosing  $F$  and  $G$ . To achieve this vertex-label balance, we may have to tweak some of the labellings in Table I; however, the tweaking must not disturb the existing edge-label balance for that labelling.

Before we proceed with the choices, we first note that if we consider the cases based on the values of  $l_1, t_1, l_2, t_2$  and exhibit the choice of labelling in each case, then we will be dealing with a large number of cases. To circumvent this, we observe that there are four distinct types of vertex-label balance achieved by the labellings in Table I. These are given in Table III.

**Table III: Balance Relation Between Vertex Labels**

Type	Labelling $\alpha_{lr}$	$v_{\alpha_{lr}}(-1)$	$v_{\alpha_{lr}}(1)$	$v_{\alpha_{lr}}(i)$	$v_{\alpha_{lr}}(-i)$
A	$\alpha_{10}, \alpha_{11}, \alpha_{20}, \alpha_{30}, \alpha_{40}$	1	1	0	0
B	$\alpha_{21}, \alpha_{43}$	1	1	1	0
C	$\alpha_{22}, \alpha_{31}, \alpha_{42}$	0	0	0	0
D	$\alpha_{23}, \alpha_{41}$	0	1	0	0

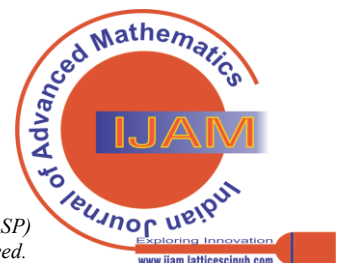
Hence rather than looking at the specific labellings to be chosen for each of the plys  $P_{t_1}^{l_1}(u_1, w)$  and  $P_{t_2}^{l_2}(u_2, w)$  We will only look at the vertex-label balance combination that arises in the two plys based on this choice. This yields only 10 cases, which we list in Table IV.

We will retain the vertex label balance in  $P_{t_1}^{l_1}(u_1, w)$  as it is, and tweak the labelling in  $P_{t_2}^{l_2}(u_2, w)$  to achieve the vertex label balance in the double uniform ply. We remark that if any two of the labels  $-1, i, -i$  are interchanged, the edge labels are unaffected; however, the vertex label balance will

shift between them. For instance, if we replace  $-1$  by  $i$  in the labelling  $\alpha_{10}$ , we get  $v_{\alpha_{10}}(1) = v_{\alpha_{10}}(i) = v_{\alpha_{10}}(-1) + 1 = v_{\alpha_{10}}(-i) + 1$ . Thus 1,1,0,0 can be easily mutated to 0,1,1,0.

**Case 2: Both Plys Have an Odd Number of Edges**

Since there are only 3 cases, we exhibit the choices of function for each ply in Table V. We will choose an existing labelling for each of the uniform plys  $P_{t_1}(u_1, w)$  and  $P_{t_2}(u_2, w)$  from the library of labellings in Tables I and II.



# On Group $A$ -Cordial Labeling of Double Uniform $(t_1^{l_1}, t_2^{l_2})$ – Ply

**Table IV: Balance of Labellings for Uniform  $t$ -ply (Case 1)**

Case	Vertex balance label for F				Vertex label balance for G				Vertex balance Label for H			
	-1	1	$i$	$-i$	-1	1	$i$	$-i$	-1	1	$i$	$-i$
AA	1	1	0	0	1	1	0	0 Replace -1 of the source by $i$ 0 1 1 0	1	1	1	0
AB	1	1	0	0	1	1	1	0 Replace -1 of source by $-i$ 0 1 1 1	0	0	0	0
AC	1	1	0	0	0	0	0	0	1	0	0	0
AD	1	1	0	0	0	1	0	0	1	1	0	0
BB	1	1	1	0	1	1	1	0 Replace -1 of source by $-i$ 0 1 1 1	0	0	1	0
BC	1	1	1	0	0	0	0	0	1	0	1	0
BD	1	1	1	0	0	1	0	0	1	1	1	0
CC	0	0	0	0	0	0	0	0	1	0	1	1
CD	0	0	0	0	0	1	0	0	0	0	0	0
DD	0	1	0	0	0	1	0	0	0	1	0	0

**Table V: Balance of Labellings for Uniform  $t$ -ply (Case 2)**

Case	Vertex balance label for F				Vertex label balance for G				Vertex and Edge label balance for H			
	-1	1	$i$	$-i$	-1	1	$i$	$-i$	-1	1	$i$	$-i$
AA'	1	1	0	0	0	1	1	0	1	1	1	0 $e_H(0) = e_H(1)$
AC	1	1	0	0	0	0	0	0	1	0	0	0 $e_H(0) = e_H(1)$
CC'	0	0	0	0	0	0	0	0	1	0	1	1 $e_H(0) = e_H(1)$

Thus, in all cases, we have obtained a group  $A$ -cordial labelling of double uniform  $(t_1^{l_1}, t_2^{l_2})$  –ply with each path of length at least 5.

### III. CONCLUSION

In this paper, we have decreased the number of cases greatly and avoided the brute force technique to prove that every double uniform  $(t_1^{l_1}, t_2^{l_2})$ -ply with each path of length at least 5 is a group  $A$ -cordial.

### DECLARATION STATEMENT

Some of the references cited are older and are noted explicitly as [5]. However, these works remain significant for the current study, as they are pioneering in their fields.

As the article's author, I must verify the accuracy of the following information after aggregating input from all authors.

- **Conflicts of Interest/ Competing Interests:** Based on my understanding, this article has no conflicts of interest.
- **Funding Support:** This article has not been funded by any organizations or agencies. This independence ensures that the research is conducted objectively and without external influence.
- **Ethical Approval and Consent to Participate:** The content of this article does not necessitate ethical approval or consent to participate with supporting documentation.
- **Data Access Statement and Material Availability:** The adequate resources of this article are publicly accessible.
- **Author's Contributions:** The authorship of this article is contributed equally to all participating individuals.

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