Indian Journal of Advanced Mathematics (IJAM)
ISSN: 2582-8932 (Online), Volume-6 Issue-1, April 2026

The Time-Long Misconceptions in Classical Mathematics

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Abstract: Many foundational rules in elementary and advanced mathematics are accepted as axiomatic, despite being introduced through heuristic or pedagogical arguments rather than strict logical or physical justification. This work undertakes a systematic analytical re-examination of several such widely accepted mathematical conventions to identify internal inconsistencies and clarify their conceptual basis. The study begins by reanalysing the four basic arithmetic operations—addition, subtraction, multiplication, and division—by reducing multiplication and division to repeated addition and subtraction. Within this framework, the conventional sign rules governing the multiplication of negative quantities are critically examined. It is shown that while the outcomes of these rules are operationally consistent, the standard logical justifications commonly provided are incomplete or internally inconsistent when interpreted in terms of direction, orientation, and repetition. Using geometric constructions and physically motivated examples, the work further examines the interpretation of signed quantities, demonstrating that positive and negative values naturally encode directionality rather than intrinsic magnitude. This perspective is extended to areas and volumes, which are typically assumed to be strictly non-negative. The analysis shows that signed areas and volumes can be meaningfully interpreted within a consistent mathematical–physical framework, thereby questioning one of the key assumptions underlying the introduction of imaginary quantities. The paper also revisits the treatment of fractional quantities, squaring operations, and dimensional comparisons, arguing that several commonly cited "paradoxes" arise from conflating quantities of different dimensions or from scale-dependent representations rather than from inherent mathematical necessity. Additionally, division by zero and infinity is reinterpreted through the lens of repeated subtraction, yielding a physically intuitive understanding of divergence and null results. Finally, the exponential function and its Taylor series expansion are examined from combinatorial and geometric perspectives, offering an alternative interpretation of the exponential constant based on dimensional arrangements rather than abstract growth alone. Overall, this work does not seek to discard established mathematical tools, but to clarify their conceptual foundations by enforcing consistency across arithmetic, geometry, and physical interpretation. The results highlight the need for greater precision in distinguishing operational rules from their underlying logical and physical meanings.

Manuscript received on 19 January 2026 | First Revised Manuscript received on 27 January 2026 | Second Revised Manuscript received on 20 March 2026 | Manuscript Accepted on 15 April 2026 | Manuscript published on 30 April 2026.

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Keywords: Mathematical Misconceptions, Negative Numbers, Imaginary Numbers, Division by Zero & Exponential Constant (e).

Nomenclature:
NLM: Newton's Laws of Motion

I. INTRODUCTION

Mathematics, at its foundation, rests upon four basic operations: addition, subtraction, multiplication, and division. Yet on closer inspection, multiplication and division can themselves be understood as extensions of addition and subtraction [1]. For instance, multiplication may be seen as repeated addition, while division reflects repeated subtraction [2]. These operations, often treated as independent and unquestionable rules, are in fact layered constructions built upon simpler principles [5].

This article begins by revisiting these fundamental operations and their logical underpinnings. Through examples and analysis, it highlights where conventional explanations in mathematics may fall short, and how specific long-accepted rules may conceal deeper inconsistencies. By re-examining multiplication and division in relation to addition and subtraction, the stage is set to question broader mathematical conventions—ranging from the treatment of negative numbers to the idea of imaginary numbers, the behaviour of fractions under squaring, and the meaning of constants such as the exponential base *e*.

$$\text{a. } x + x + x + x \ldots\ldots\ldots\ldots..y \text{ times} = xy \quad \ldots \quad (1)$$
$$\text{b. } y + y + y + y \quad \ldots\ldots\ldots\ldots..x \text{ times} = xy \quad \ldots \quad (2)$$

So, multiplication is basically summation only.

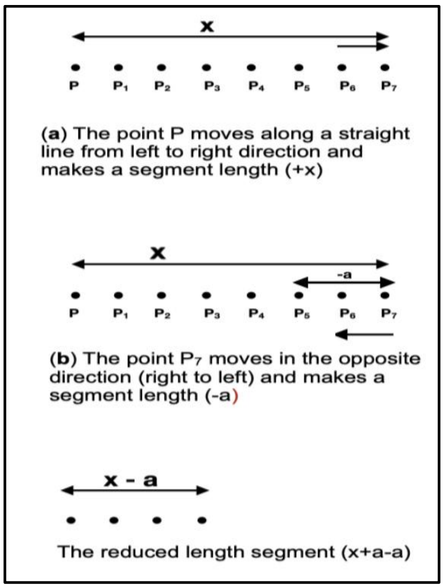
The operation of division can be understood as repeated subtraction. For example, consider dividing 20 by 4. Here, 20 is the dividend, 4 is the divisor, and the result—the quotient—is 5. This quotient emerges from successive subtraction of the divisor from the dividend, as shown below:

1. Step 1 - (20 - 4) = 16
2. Step 2 - (16 - 4) = 12
3. Step 3 - (12 - 4) = 8
4. Step 4 - (8 - 4) = 4
5. Step 5 − (4 − 4) = 0 ... (3)

Thus, the operation of division is nothing more than repeated subtraction, carried out until the result reaches zero. The quotient is simply the number of subtractions steps performed. In the earlier example, subtracting 4 from 20 five times reduces the dividend to zero; therefore, the quotient is 5.

If this idea is analyzed more broadly, we can see that every mathematical operation—whether differentiation, integration, squaring, exponentiation, extracting roots, logarithms, factorials, or even metric operations—can ultimately be reduced to the fundamental processes of addition or subtraction.

With this perspective in place, it is also essential to revisit the meaning of the mathematical signs (+) and (–). These are not absolute properties of numbers but rather conventional symbols that represent the direction of change, of transition, or of orientation. This can be illustrated schematically, as shown in Figure 1, where the growth and decline of a straight line are depicted.

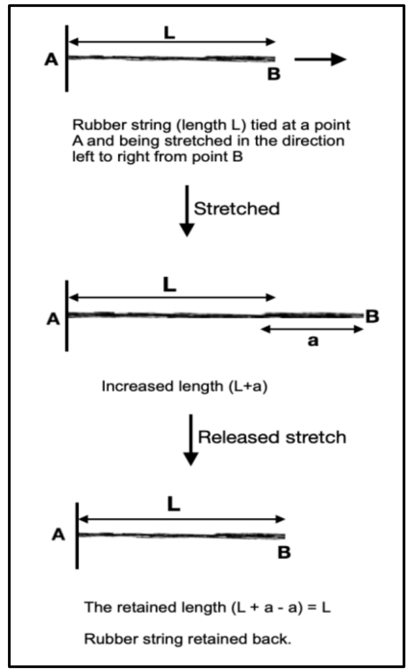


[Fig.1: The Schematic Presentation of (+) and (-) as the Index of Growth and de-Growth]

In Figure 1, the (+) sign is traditionally associated with growth, while the (–) sign is linked with de-growth. Yet this assignment is purely conventional. The symbols could easily be reversed—associating (+) with de-growth and (–) with growth—without altering the underlying reality.

This arbitrariness is evident in real-world contexts. For instance, in banking operations, when a person deposits money into an account, the funds flow from the individual to the bank. By convention, this is treated as positive. Conversely, when money is withdrawn, the flow is from the bank to the individual, and this is recorded as a negative amount.

A similar example can be drawn from physics. Consider a flexible rubber string being stretched from one end. As the string extends, its length increases; this change is labelled positive, with the direction of extension taken to be left to right. When the force is released, the string contracts in the opposite direction (right to left), and this change is labelled negative. If the extended length of the string is x, and it shrinks by y units, the remaining length is expressed as $(x - y)$. In both examples, it is the orientation or direction of change—not any inherent property of numbers—that dictates whether we assign a (+) or (–) sign.



[Fig.2: Direction of Elongation and Shrinking Dictates (+) and (-)]

In multiplication operations, the following rules are followed:

a. (+) x (+) = (+)
b. (+) x (-) = (-)
c. (-) x (-) = (+)

However, to note here that if [(+) x (+) = (+)] is considered to be true, it does not logically follow that [(-) x (-)] = (+)]. It should be, for very logical grounds, [(-) x (-)] = (-)] only. Mathematics cannot explain why, [(+) x (-)] = (-)]? The following mathematical operations are worth noting:

For example (-x) is being multiplied with (-y),

$$\text{Conventional mathematics: } [(-x)x\,(-y)] = +xy \quad \dots \quad (4)$$

$$\text{Logical mathematics approach: } [(-x)x\,(-y)] = -[(+x)x\,(+y)] = -xy \quad \dots \quad (5)$$

For example, when (+x) is being multiplied by (-y), (following conventional mathematics)

$$[(+x)x\,(-y)] = -[-xy] \quad \dots \quad (6)$$

So conventional mathematics cannot explain why [(+) x (-)] = minus. However, through the new concept of this article as described below:

For example, when one multiplies (+ 5) by (+3), this can be stated in two ways,

i) (+5) is being made 3 times and the result is (5 + 5 +5) =15
ii) (+3) is being made 5 times and the result is (3 + 3 + 3 + 3 +3 +3) = 15

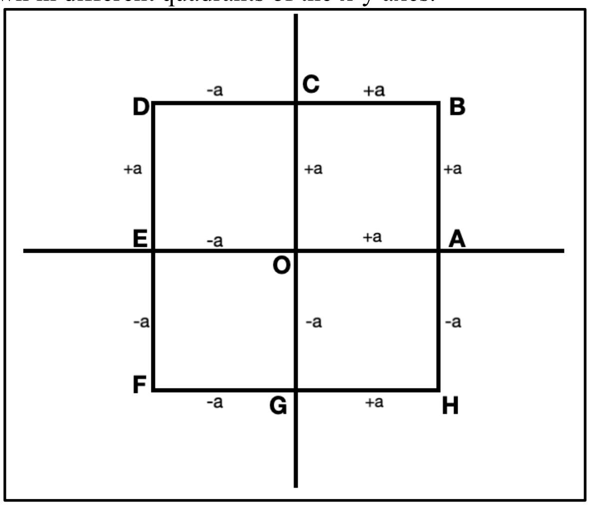
When it comes to multiplication, consider the example of (+5) × (-3).

Indian Journal of Advanced Mathematics (IJAM)
ISSN: 2582-8932 (Online), Volume-6 Issue-1, April 2026

This can be interpreted as taking (–3) five times, which gives (–3 – 3 – 3 – 3 – 3) = –15. What cannot be justified, however, is the reverse interpretation: taking (+5) a negative number of times. Counting is always inherently positive, so the idea of repeating something a "negative number of times" has no real meaning.

Similarly, if (+3) is multiplied by (–5), the operation should be understood as taking (–5) three times: (–5 – 5 – 5) = –15. Through this reasoning, the conventional rule that (+) × (–) = (–) can be rationally justified, not as an abstract law but as a logical outcome of repeated addition and subtraction.

In Figure 3 below, the areas of four-square numbers have been shown in different quadrants of the x-y axes.



[Fig.3: Positioning of four Squares in 4 Different Quadrants and Their Areas Being Linked to the Orientation of the Squares and the Direction of Formation of the Lengths of the Squares]

In Figure 3, the origin O is taken as the starting point. Any point moving from the origin to the right along the x-axis (or parallel to it) traces a segment length that is considered positive. Conversely, movement from the origin to the left along the x-axis is treated as negative. Similarly, movement upward along the y-axis (or parallel to it) is considered positive, while movement downward is regarded as negative.

The figure consists of four squares: OABC, OECD, OAHG, and OEFG. Each side of these squares—OA, AB, BC, OC, OE, DE, CD, EF, OG, OF, GH, and AH—has the same magnitude, denoted by *a*. What differs is not the length itself but the sign assigned to it, depending on the direction of movement from the origin.

The lengths,

$$OA = BC = (+a) \quad ... \quad (7)$$

$$OC = DE = (+a) \quad ... \quad (8)$$

$$DC = FG = (-a) \quad ... \quad (9)$$

$$OC = (+a), OG = -a \quad ... \quad (10)$$

The areas of the four numbers of squares are:

$$\text{Area OABC} = (+a)x(+a) = a^2 \quad ... \quad (11)$$

$$\text{Area OCDE} = (+a)x(-a) = -a^2 \quad ... \quad (12)$$

$$\text{Area OGHA} = (+a)x(-a) = -a^2 \quad ... \quad (13)$$

$$\text{Area OGFE} = (-a)x(-a) = -a^2 \quad ... \quad (14)$$

From the above exercise, it follows that areas can be regarded as positive or negative, depending on the sign of their sides. When the area is positive [as expressed in equation (11)], each side of the square is taken as +a. However, when the area is considered harmful, two cases arise, as outlined in equations (12)- (14).

i) One side of the square is +a and the other side is -a.
ii) Or, both the sides are –only.

We are being habituated in writing, if the area of a square is $x^{2,}$ then,
x^2 = 25 (for example), then

$$x = +5 \text{ or } -5 \quad ... \quad (15)$$

It should be written as,

$$x = +5 \text{ only} \quad ... \quad (16)$$

When

$$x2 = -25, \text{then}$$

$$x = +5 \text{ and } -5 \text{ (one side } +5 \text{ and another } -5) \quad ... \quad (17)$$

or,

$$x = -5 \quad ... \quad (18)$$

For the calculations of volume (which is the cube of a length), the volumes are considered to be positive only. If the figure 3 above is being extended to a 3D space, the same conclusions as for the areas would be obtained.

If the volume is being x^3,

i) When x^3 is positive, the three roots would be +x, +x and +x only.
ii) When x^3 is negative, the three roots would be -x, -x and -x or +x, -x and -x or +x, +x and -x only.

A. Natural Question Arises: How can Volume be Negative?

This can be illustrated by a water tank fitted with two pipes. Suppose the tank initially contains water up to a certain height. Through one pipe, water enters at a definite rate, while through the other, water drains out at a definite rate. At any instant, the net volume of water in the tank equals the sum of the inflow and outflow contributions. If the inflow dominates, the volume increases and is taken as positive; if the outflow dominates, the volume decreases and may be regarded as negative.

Volume of water in the tank at any instant = (original volume of water + volume of water entered at the tank at that instant) + (volume of water being drained out at that instant) (19). If the volume of water entering the tank is taken as positive, then the volume of water draining out must be considered harmful, since it represents flow in the opposite direction. Accordingly, in equation (19), the first term (initial volume) is positive, the second term (inflow) is positive, and the third term (outflow) is negative.

If the inflow rate exceeds the outflow rate, the water level in the tank will rise over time. Conversely, if the outflow rate is greater, the water level will fall over time. This provides a clear physical meaning to the concept of negative volume.

For instance, suppose at a given instant the original volume of water in the tank is 100 L, the volume of water entering is 40 L, and the volume of water draining out is 60 L. In this case, the net volume of water in the tank would be:

$$100 + 40 – 60 = 80 \text{ L.}$$

The net volume of water in the tank is (100 + 40 - 60) = 80 [as per equation (19)]. This indicates that the volume may be negative as well.

This concept extends naturally to areas as well. In biological systems, cells continuously undergo two opposing processes: cell fusion and cell division. During fusion, two cells combine to form a single larger cell. This reduces the total surface area, which can be considered as a negative contribution. In contrast, during cell division, a single cell divides into two, thereby increasing the total surface area—a positive contribution [4].

The same logic applies to the agglomeration and de-agglomeration of fine particles in a substance: agglomeration reduces surface area, while de-agglomeration increases it. Thus, at any instant, the net surface area of a system is determined by the balance between these two opposing processes—growth and shrinkage—mirroring the idea of positive and negative contributions.

Total surface area of the cells = (Surface area of the cells + surface area generated due to cell division + the surface area diminution due to the cell fusions) (19a)

If the rate of cell division exceeds the rate of cell fusion, the total surface area of the cells will increase, dominated by the positive contribution of division. Conversely, if the fusion rate is higher, the total surface area will decrease, reflecting the negative contribution of fusion. This demonstrates, in a tangible biological context, that areas can also be meaningfully regarded as positive or negative in our universe.

The concepts discussed above form what may be called a *tripartite principle*. They are tripartite in the sense that they apply simultaneously to mathematics, geometry, and physics. A formula or hypothesis should not be considered valid if it holds only within the narrow framework of one discipline. Rather, any robust hypothesis must remain consistent across all three perspectives—mathematics, physics, and topology—so that the underlying logic is universally coherent.

Once it is established that areas can also take negative values, the very foundation of the concept of imaginary numbers begins to fade. The construct of imaginary numbers in mathematics has historically been built on a set of assumptions—chief among them the belief that specific quantities (such as negative areas or square roots of negative numbers) cannot exist within the realm of real numbers.

i) Areas cannot be negative

ii) Square rooting of a negative number is not possible.

As established earlier in this article, the two key assumptions underlying the notion of imaginary numbers are invalid. Accordingly, the square root of (–1)—conventionally denoted as *i*—need not imply an imaginary construct. Instead, it has two possible interpretations: either both roots are -1, or one root is +1 while the other is -1.

Once the very basis of imaginary numbers is shown to be illogical, the concept of complex numbers loses its special meaning. In reality, all numbers are real, and mathematics should move beyond the artificial construct of imaginary numbers altogether.

1. Square or cube of a decimal number or decimal variable.

In conventional mathematics, it is considered that if a fraction is being squared, its magnitude or value decreases. If, for example, a fraction 0.2 is squared, the value would be 0.04, and hence it is considered to be less than the original fraction. These types of thoughts are incorrect, as will be established here with logical arguments.

First, it should be understood that fractions are always expressed as ratios. If there are 100 apples in a box and a person picks up, for example, 0.2 of the apples. Then the number of apples the person picks up is (100 × 0.2) = 20 apples. Now, if this fraction 0.2 is squared, it attains a value of 0.04. Then the number of apples would be (100 × 0.04) = 4 apples. However, there does lie a misconception in mathematics in this event. Whenever one is squaring a fraction, the whole expression must be squared. While the total number of apples was 100, this needs to be considered as (100)2 when squaring the fraction. So the total number of apples would be 10000, and 0.04 fraction of that would be (10000 x 0.04) = 400. So, when squaring a fraction, its magnitude increases. In case of cubing the fraction, so that 0.2 becomes 0.0008, and the whole thing becomes (100)3 = (1000000), the number of apples picked up to be considered is (100000 x 0.0008) = 800. So this value of the cube of the function stands to be higher than the square of the fraction, which is 400. So one can conclude, going against the conventional mathematics,

[Cube of a fraction] > [square of a fraction] > [the fraction itself] (20)

When applied to length or distance, the above concept extends into a different horizon. Length is measured in physical units—kilometres, meters, centimetres, millimetres, and so on. If, for example, a length of 0.2 cm is considered, its square is 0.04 cm^2. At first glance, one might conclude that the square is “smaller” than the original value. But this comparison is invalid, because it equates quantities of different dimensions: a one-dimensional length with a two-dimensional area. Length, location, and volume exist in fundamentally different dimensional spaces, and comparing them directly as if they were of the same order is mathematically inconsistent.

Even if such cross-dimensional comparisons are accepted for the sake of argument, another perspective arises when we shift the scale of measurement. Consider again 0.2 cm. This is equivalent to 20 mm, since

10 mm = 1 cm. Squaring 20 mm yields 400 mm², which is not less than 20 mm. Likewise, 0.1 square meters is 0.01 m², apparently less than 0.1. But when expressed as 10 cm, its square is 100 cm²—greater than 10 cm.

Thus, the conventional belief that "squaring a fractional number or variable always produces a smaller result" is not a valid conclusion. Dimensionality and scale matter; once these are properly accounted for, the claim collapses.

II. NON–EXISTENCE OF FRACTIONS

A crucial conceptual point about length or distance is that they cannot, in essence, be fractional. Consider a distance of 1.23456 km. At first glance, this appears fractional; however, if we reduce the unit of measurement, the value can always be expressed as an integer. The exact distance is 1234.56 meters, 123456 centimetres, or 1234560 millimetres. With each downgrade of scale, the apparent fraction dissolves, and the distance becomes an integer count of the chosen unit. [3]

This demonstrates that "fractional length" is not an inherent property of distance itself but instead an artefact of the unit system we impose. The continuity of length is absolute; fractionality arises only from the granularity of our measurement.

1.23456 km = 12.3456 hectometer = 123.456 decameter = 1234.56 meter = 12345.6 decimeter = 123456 cm.

For any fractional distance, if the scale of measurement is continually downgraded, it will eventually appear as a whole number. In principle, by refining the scale further, one would approach the smallest possible length of the universe—a fundamental unit of distance. Under such circumstances, any length, whether initially written as a fraction or as a whole number, could be expressed as an integral multiple of this smallest length. [3]

Thus, in tangible physical terms, no truly fractional length exists. Fractions emerge only as a convenience of mathematical representation, not as a property of nature itself. They serve as tools for calculation, but the concept of a "fractional length" is conceptually fragile, arising from abstraction rather than physical reality.

A. Mathematical Expressions Beginning with a Minus Sign Before a Bracket, and the Convention of Mathematics

In mathematical operations, for a mathematical operation as below is being handled in the following way,
Mathematical Expression:

$$[x - (5 - 3)] = [x - 5 + 3] = (x - 2) \quad ... \quad (21)$$

One can argue that [(-) x (-)] = (+), since -3 is written as +3 in equation (21).

To note here that the expression is to be written in the following fashion,

$$[x - (5 - 3)] = [x - 2] = (x - 2) \quad ... \quad (22)$$

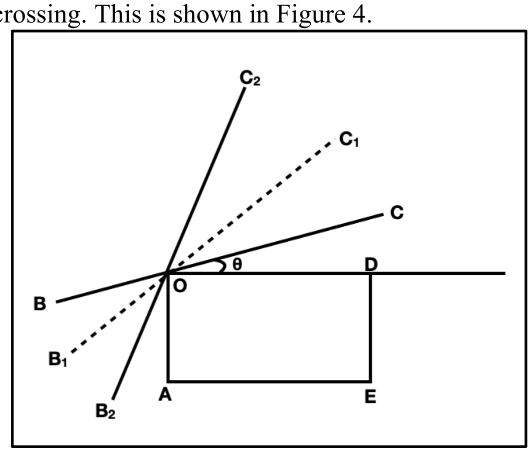
The concept of [(-) x (-)] = (+) is not all involved here; in mathematical operations, people do this type of manipulation to complete the mathematical operation quickly.

As discussed earlier, the concepts of (+) and (–) are rooted in direction or orientation. But there is another key dimension: they are also *relative*. Consider a wooden stick 10 meters long, cut into two pieces—one of 6 meters and the other of 4 meters. From the perspective of the 4-meter piece, the remaining 6 meters can be viewed as negative, since its own length was obtained by subtracting 6 from 10: (10 – 6) = 4. Conversely, from the perspective of the 6-meter piece, the 4 meters is negative, because its own length was obtained by subtracting 4 from 10: (10 – 4) = 6.

This relativity reinforces the idea that positive and negative are not absolute properties, but context-dependent. Based on this reasoning, it is both logical and justified to accept that negative areas and volumes are meaningful. One cannot hold as sacrosanct the claim that negative values for such quantities are forbidden. In mathematics, both positive and negative areas and volumes must be acknowledged—there is no choice or exception.

B. Physical Significance of the Constant in the Straight-Line Equation in the form (y = mx +c)

A constant in mathematics is an entity (which could be anything, a line segment, an area or a volume) that does not change its position or shape during a physical process occurring. A very concrete practical example of the mathematical equation of a straight line in the form of (y =mx + c), [where m is the slope and c is the intercept], is the railway level crossing. This is shown in Figure 4.



[Fig.4: A Railway Level Crossing Resembles [y = mx +c]]

In Figure 4, the length OA remains fixed. Its orientation and shape do not change with time. What changes instead is the orientation of the straight-line BOC. Imagine a railway crossing: while a train is passing, the barrier rests along the straight-line OD, making its angle with the horizontal, θ, equal to zero. Once the train has passed and the barrier is lifted, the line BOC slowly rotates about point O, sometimes upward and sometimes downward, continually changing the value of θ. In this representation, the line A corresponds to *y* in the equation $y = mx + c$, where the tangent of angle COD (= θ) gives the slope or gradient *m*, and the Fixed-length OA represents the constant *c*.

For the constancy of area, consider the example of a fluid flowing through a pipe of fixed length and cross-sectional area. If the pressure on the fluid is increased, the flow rate (the volume of water exiting the pipe per unit time) increases. Yet at any instant, the volume of fluid contained within the pipe remains constant because both the pipe's length and cross-section are fixed.

The same reasoning applies to the constancy of volume. If the pressure is kept constant, the volume of fluid eluted from the pipe per unit time also remains steady.

C. Dividing a Variable by Zero or Infinity and the Underlying Concepts

In mathematics, it is conventionally held that dividing any variable by zero yields infinity. This can be better understood by revisiting the process of division as explained earlier in this article: division is simply repeatedly subtracting the divisor from the dividend until the remainder is zero. The number of subtraction steps taken corresponds to the quotient.

Now, in (x/0)

Step 1. x - 0 = x

Step 2. x - 0 = x

Step 3. x - 0 = x

Step 4 x - 0 = x

………………………………

………………………………..

If this is continued for an infinite number of steps, the result would never be zero. Hence,

$$\left(\frac{x}{0}\right) = \infty \quad \ldots \quad (23)$$

Now, when the case of (x/∞) is being considered, it is to note that subtracting ∞ from x, that is, (x-∞), is not possible since an infinite number can never be deducted from a finite number, since the value of the endless number is not known to us at all. Thus, the number of steps would be zero (no subtraction step is possible). Hence,

$$\left(\frac{x}{\infty}\right) = 0 \quad \ldots \quad (24)$$

D. The Physical Significance of the Exponential Factor and the Problem in the Conventional Taylor's Power Series of ex

The e parameter is a fundamental mathematical constant with a value of 2.71828... However, the e^x-function (popularly known as the exponential function) has been expressed in mathematics by a power series (called Taylor's series) as follows,

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots + \frac{x^n}{n!} + \cdots$$

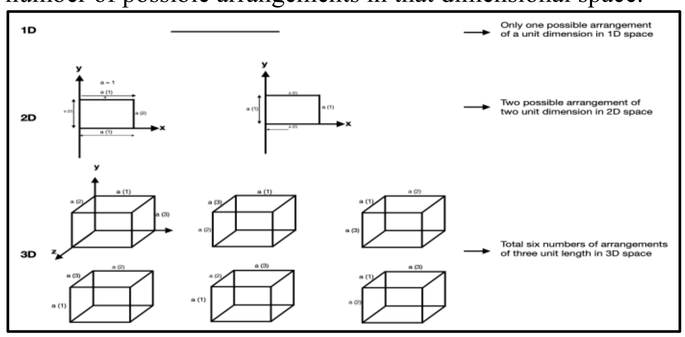
....Equation 26

Let us now focus on the case where the variable has magnitude 1, or $x = 1$. In the power series expansion, the first term is always 1, regardless of the value of x. The second term is $(x / 1!)$, which in this case becomes $(1 / 1!) = 1$. The physical meaning of the numerator (1) is that it represents a unit length in one-dimensional space. The denominator (1! = 1) signifies the number of possible arrangements of that unit length in 1D space, which is only one.

Moving to the third term of the series, the numerator is 1^2, which represents two-unit lengths forming a two-dimensional unit square. The denominator (2! = 2) corresponds to the number of possible ways to arrange these two-unit lengths in 2D space.

For the fourth term, the reasoning extends into three dimensions. The numerator 1^3 represents three-unit lengths forming a unit cube in 3D space. The denominator (3! = 6) gives the number of possible arrangements of these three-unit lengths in three-dimensional space.

This interpretation, illustrated schematically in Figure 5, shows how each successive term of the series carries geometric and physical significance: the numerator encodes dimensional units, while the factorial in the denominator encodes the number of possible arrangements in that dimensional space.



[Fig.5: The Number of Possible Arrangements of unit Lengths in 1D, 2D & 3D Space]

So, all the terms of the power series of equation (26) represent (except the first term), the following ratio,

Ratio of the numerator and the denominator of the power series (excluding the 1st term)

= (The number of unit length variables of an n-D space)/ (number of possible arrangements of the n numbers of unit length in the n-D space) (27)

As the value of n increases from 1 to 2 to 3, 4, and so on, the magnitude of the ratio in equation (27) decreases steadily. This happens because the number of possible arrangements in n-dimensional space grows factorially: 1! = 1, 2! = 2, 3! = 6, 4! = 24, 5! = 120, 6! = 720, and so on, up to $n!$ While the dimension n increases only linearly—by a single unit each step—the number of possible arrangements grows explosively. This disparity is directly reflected in the decreasing values of successive terms in the power series.

The essential feature of this series is that as the number of variables (n) increases, the number of possible arrangements in n-dimensional space also increases without bound. Traditionally, the Taylor series (equation 26) has been described as an "exponential increasing/decreasing series," with its exponential character tied

Retrieval Number:100.1/ijam.A123606010426
DOI: 10.54105/ijam.A1236.06010426
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to the variable *x*. Yet this interpretation is misleading. The actual dependence is not on *x* but on the number of variables, *n*.

In this light, the Taylor series retains clear physical meaning only when *x = 1* or *x = 2*. Beyond this—when *x* takes values of 3, 4, 5, 6, and so on—the conventional interpretation loses coherence, as demonstrated in Table 1.

Table I: The Values of the Different Terms of the Power Series of Equation (26) as a Function of Differing Values of x (x = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10)

Exponential function e^x	The value of the 1st first term of the power series	Value of the 2nd term	Value of the 3rd term	Value of the 4th term	Value of the 5th in terms	Value of the 5th term	Value of the 6th term	Value of the 7th term	Value of the 8th term	Value of the 9th term	Value of the 10th term	The total of the nth term of equation (25)
e^1	1	1	0.5	0.1666	0.0416	0.0083	0.00138	0.00018	too small			2.71828.
e^2	1	2	2	1.3333	0.6666	0.2666	0.0888	0.02539	0.00635	0.00141	0.00082	7.389056.
e^3	1	3	4.5	4.5	3.375	2.025	1.0125	0.4339	0.1627	0.05424	0.01627	20.08549.
e^4	1	4	8	10.6666	10.6666	8.5333	5.6888	3.2508	1.6253	0.7224	0.2889	54.59800.
e^5	1	5	12.5	20.8333	26.0416	26.0416	21.217	15.5009	9.6881	5.3822	2.6911	148.4131.
e^6	1	6	18	36	54	64.8	64.8	55.5428	41.6571	27.7714	16.6628	403.4287
e^7	1	7	24.5	57.1666	100.0416	140.0583	163.403	163.403	142.976	111.2037	77.8426	1096.633
e^8	1	8	32	85.333	170.6666	273.0666	364.08	416.105	416.1015	369.868	295.894	2980.9579
e^9	1	9	40.5	121.5	273.375	492.07	738.1125	949.007	1067.627	1067.627	960.86	8103.0839
e^{10}	1	10	50	166.6666	416.6666	833.3333	1388.88	1984.16	2480.158	2755.739	2755.71	22026.466

To make the Taylor series valid for values of *x = 3, 4, 5, 6, ..., n*, the variable *x* should be reinterpreted. It should not be regarded as adding new spatial dimensions, but rather as scaling the unit length itself. Thus, when passing from *x = 1* to *x = 3*, the unit length is effectively tripled; when *x = 4*, the unit length becomes four times larger, and so on. Under this view, the number of possible arrangements of *x* lengths in space becomes independent of the actual value of *x*.

For example, consider a cube. Regardless of whether its side length is 1 mm or 1 km, the number of possible arrangements of its three sides in space remains fixed at 3! = 6. The size of the cube does not affect the factorial count of arrangements.

This interpretation also clarifies the transition from *x = 1* to *x = 2*. When *x* increases from 1 to 2, it may be understood as doubling the unit length, so that a single unit of length 2 is treated as the new unit. Yet the number of possible arrangements of these two lengths remains 2! = 2. Hence, the power series of equation (26) follows the same structural pattern as for *x = 1*.

For values of *x* greater than 2, however, the series no longer retains the behaviour of a pure exponential. Instead of diverging monotonically to infinity in one direction and converging smoothly toward zero in the other, the series exhibits a maximum before decreasing. This diverges from the defining property of an actual exponential function.

Thus, the celebrated *e*-function holds its ideal physical significance only for *x = 1*. At *x = 2*, the exponential form is barely retained, though it is not entirely appropriate even here. For higher values of *x*, the traditional exponential interpretation breaks down altogether.

In mathematics, the term *exponential* [3] is generally defined as a function whose magnitude depends on the current value of *x*, with *x* appearing in the exponent of an expression. For instance, in the function 2^x, the value of the function is entirely determined by *x*. If *x* is replaced by 2x, the function becomes 22x, which is proportionally larger than its previous value.

A key characteristic of exponential functions is their power-law growth relative to scaling of *x*: if the variable doubles, the function is squared; if it triples, the function is cubed; if it quadruples, the function is raised to the fourth power. Importantly, there is no requirement that the base must be *e*; any positive real number can serve as the base of an exponential function. This is illustrated in Tables 2 and 3 below.

By contrast, not every function responds in this way to changes in *x*. For example, in the function *y = f(x) = (1/x + c)*, where *c* is a constant, increasing *x* does not necessarily increase the value of *y*. Such behaviour distinguishes ordinary functional dependence from exponential dependence.

Table II: The Change in the Value of a Function 2xas a Function of the Value of the Variable x

Current Value of the Variable x	Value of the Function 2^x	Ratio of the Current Value of x to the Original Value of x	Effect on the Value of the Function 2^x
1	2	-	$(2^1)^1$
2	4	2	$(2^1)^2$
3	8	3	$(2^1)^3$
4	16	4	$(2^1)^4$
5	32	5	$(2^1)^5$

Table III: The Change in the Value of a Function 3xas a Function of the Value of the Variable x

Current Value of the Variable x	Value of the Function 3^x	Ratio of the Current Value of x to the Original Value of x	Effect on the Value of the Function 3^x
1	3	-	$(3^1)^1$
2	9	2	$(3^1)^2$
3	27	3	$(3^1)^3$
4	81	4	$(3^1)^4$
5	243	5	$(3^1)^5$

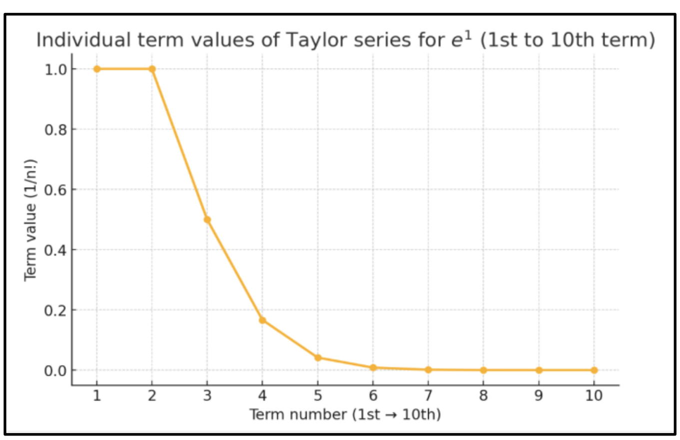
For a mathematical function such as $y = f(x) = (1/x + c)$, where c is a constant, the value of y does not necessarily increase as x increases. This function is therefore not exponential, for two clear reasons:

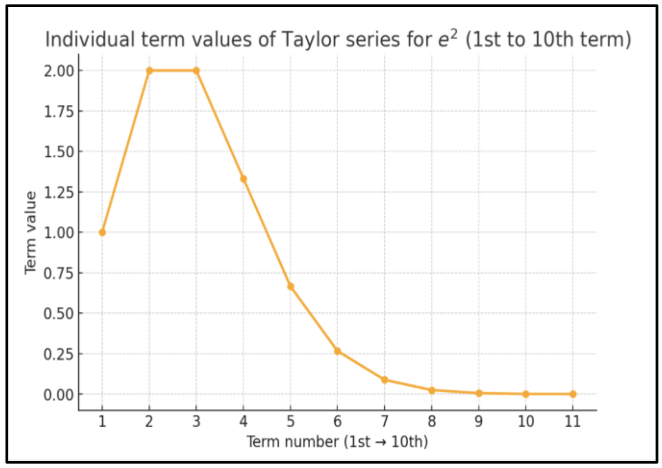
(i) x does not appear in the exponent, and

(ii) the value of the function does not consistently increase with x.

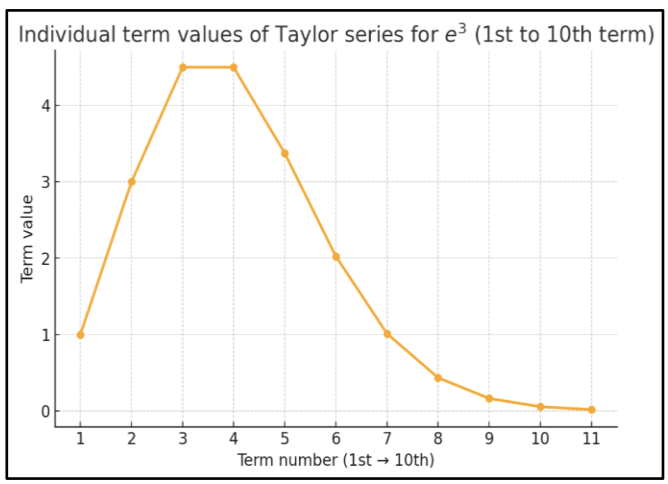
The plots of e^x (based on the data in Table 1) versus x are shown in Figures 6 through 10, corresponding to the term-by-term expansion (1st term, 2nd term, 3rd term … nth term) of the Taylor series for values of $x = 1, 2, 3, 4 \ldots 10$. Figure 11 shows typical plots of e^x, 2^x, and 3^x for different values of x, where each curve represents the sum of the power series for that value of x.

The character of these curves substantiates the analysis above. Except for the e^1 curve, none of the other exponential functions truly reflects the physical significance of the Taylor series. That significance lies in the ratio of the "number of dimensions of an n-dimensional space" to the "possible number of their combinatorial arrangements." This ratio diminishes steadily as x increases, tending toward vanishingly small values as x becomes large.





[Fig.6a & 6b: Plot of E1and E2functions Up to 10th Terms of the Taylor Series]



[Fig.7: Plot of e3 Functions up to the 10th Terms of the Taylor Series]

Retrieval Number:100.1/ijam.A123606010426
DOI: 10.54105/ijam.A1236.06010426
Journal Website: www.ijam.latticescipub.com

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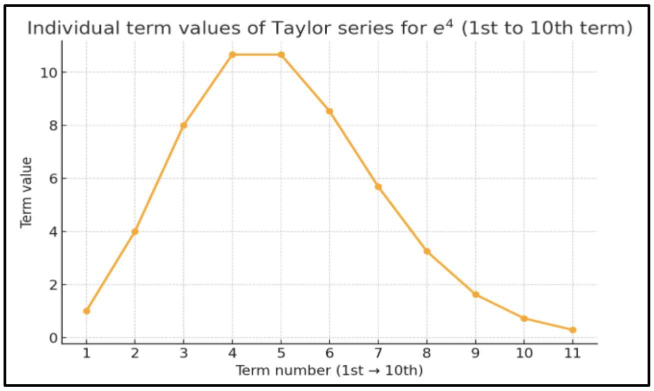
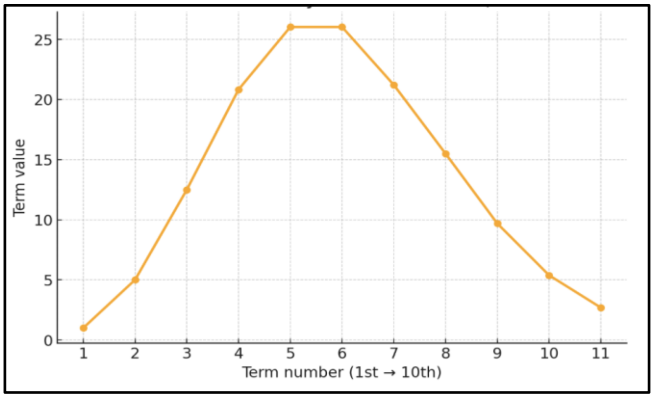
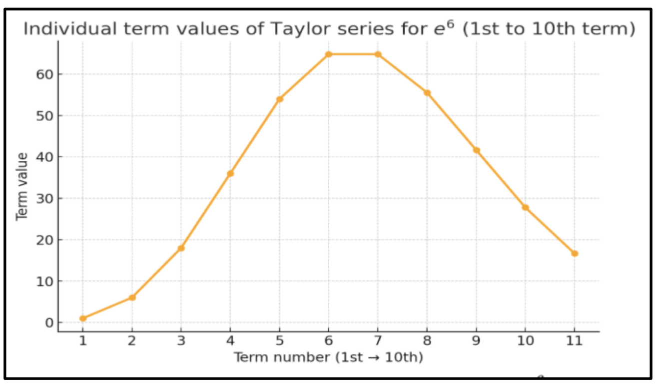
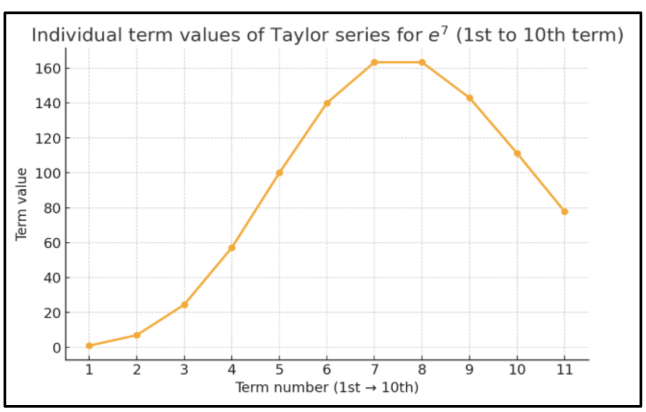


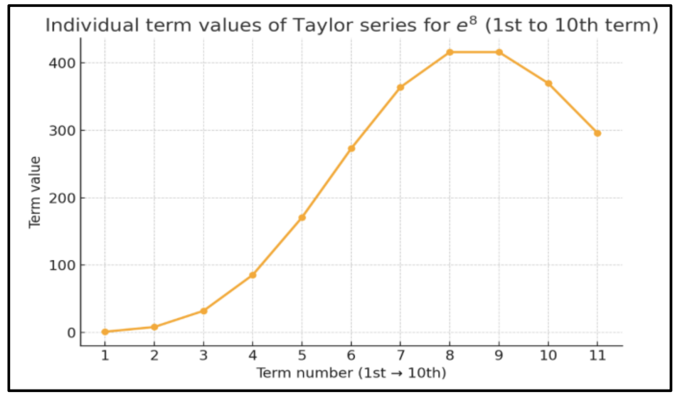


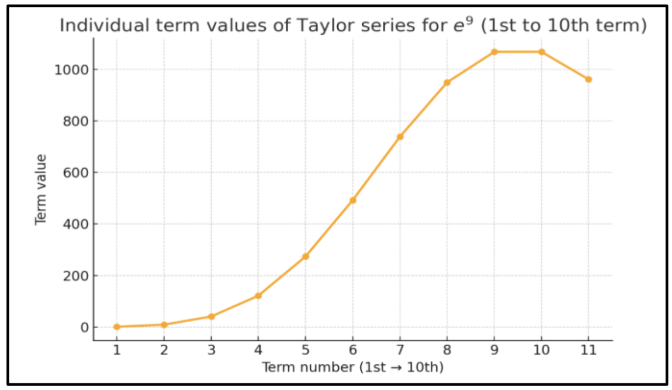
[Fig.8a & 8b: Plot of e4 and e5 Functions up to 10th Terms of the Taylor Series]

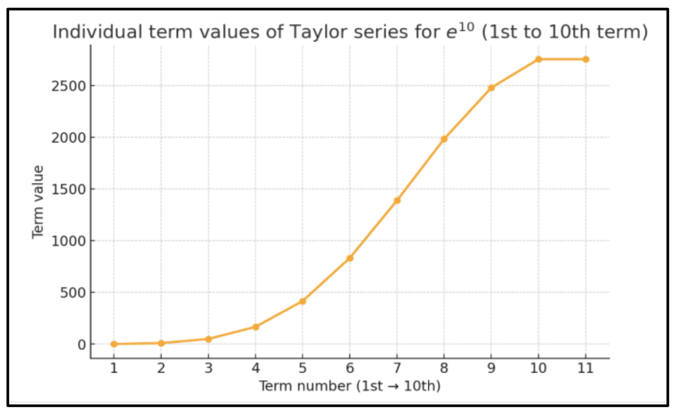




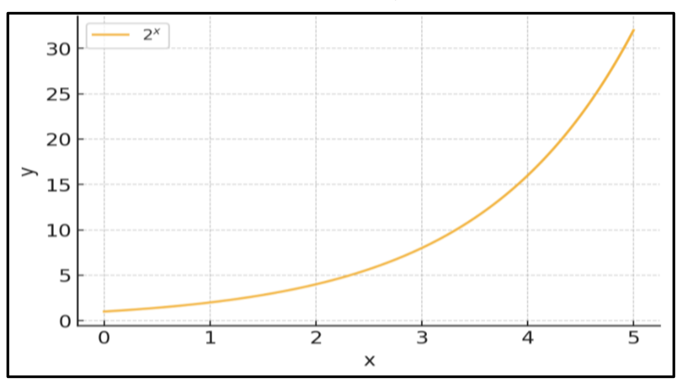
[Fig.9a & 9b: Plot of e6 and e7 functions up to 10th terms of the Taylor series]

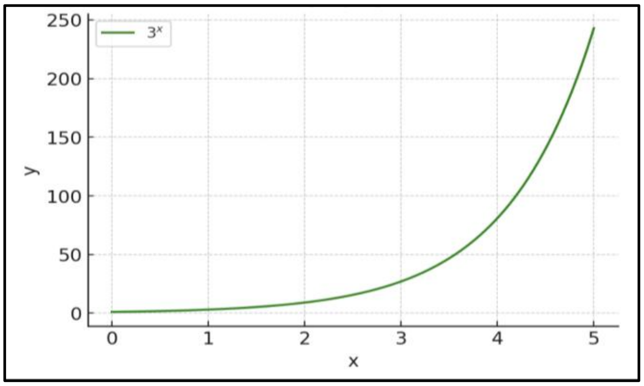


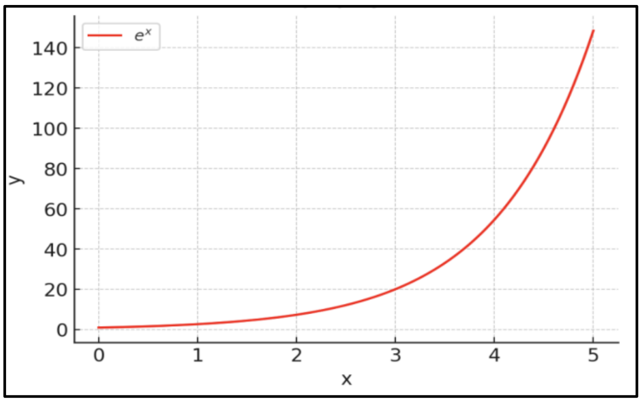




[Fig.10a, 10b & 10c: Plot of e8, e9 and e10 Functions up to 10th Terms of the Taylor Series]



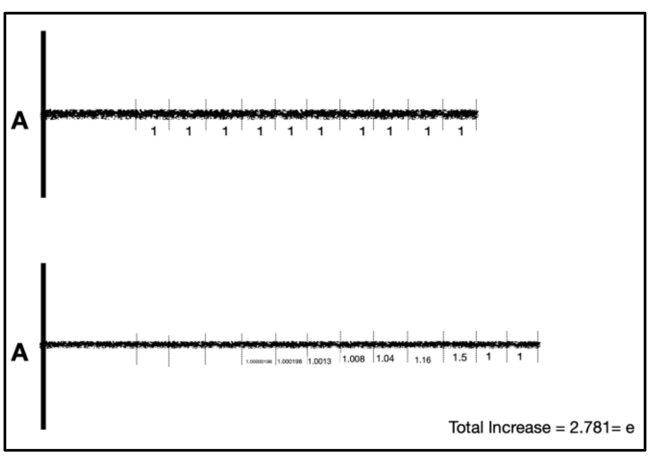




[Fig.11a, 11b & 11c: Typical Plots of 2x, 3x, and ex for the Different Values of x]

The physical significance of the *e*-function is often described in textbooks and scientific literature through examples such as continuous compounding in a bank account, population growth, bacterial multiplication, or radioactive decay. While these examples are mathematically valid, they are not fully convincing in helping learners *see* and *grasp* what exponential growth or decay truly means. They remain abstract, concealing the phenomenon's physical essence [6].

In Figure 12, a more tangible visualization is proposed. Consider a nylon string of length AB = L, divided into *n* smaller segments. Suppose end A is held fixed, and a force F is applied at the free end B.



[Fig.12: The Elongation of a Wire and the Physical Concept of Taylor's Power Series of e1]

The first small unit will be elongated the most; the second unit will be elongated less than the 1st unit. The said elongation will follow a pattern like,

1st unit > 2nd unit > 3rd unit > 4th unit > 5th unit >……………………………………..> nth unit

Or, from the opposite direction,

nth unit < (n-1) unit < (n-2) unit < (n-3) unit < (n-4) unit< ……………………………….< 1st unit

On one side, it increases exponentially (from B to A), and on the other hand, it decreases exponentially, so the growth becomes vanishingly small.

III. PHYSICAL SIGNIFICANCE OF LOGARITHM

The physical significance of the logarithmic function may now be considered. Take the base-10 logarithm as an example. We know that log (10) = 1, log (100) = 2, log (1000) = 3, and log (10000) = 4.

The logarithm can be interpreted as a *dimension-squeezing function*. [5] For instance, the number 10 is composed of ten-unit blocks, each with a value of 1. When we take log (10), the result is 1. This means that the collection of 10 units has been "compressed" into a single value. In effect, each of the 10 units is squeezed by a factor of 10, so that 1 becomes 0.1, and the sum of all squeezed units is 10 × 0.1 = 1.

The same reasoning extends to larger numbers. The number 10000 is made up of 10,000-unit blocks. Its logarithm is 4, meaning that the 10,000 units have been squeezed down to 4. In this case, the squeezing factor is 2500, so each unit is reduced by a factor of 1/2500 = 0.0004. Multiplying across all 10,000 units yields 10,000 × 0.0004 = 4, consistent with log(10000) = 4.

In this way, a logarithmic function can be understood as a systematic reduction of scale: a process that compresses large magnitudes into smaller, more manageable numbers while preserving proportional meaning. This is the tangible physical significance of logarithms.

IV. CONCLUSION

This article challenges the very foundations on which much of modern mathematics rests. The traditional rule [(–) × (–) = (+)] is not just a small slip of logic—it is the seed from which entire branches of misleading constructs have grown. Imaginary numbers, complex numbers, and the elaborate theories built around them are revealed here as unnecessary illusions.

Signs, (+) and (–), are not absolute truths. They are markers of direction—growth and de-growth—defined only in relation to one another. Once this relativity is acknowledged, the supposed impossibility of negative areas and volumes collapses; both must be admitted as real and tangible features of mathematics and the physical world.

Retrieval Number:100.1/ijam.A123606010426
DOI: 10.54105/ijam.A1236.06010426
Journal Website: www.ijam.latticescipub.com

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This work has also shown that the exponential function *e^x* and its Taylor series are not universally valid. Their physical meaning holds firmly only at *x = 1*, weakly at *x = 2*, and disintegrates beyond that. What has been passed off as "exponential law" is in fact a narrow truth overstretched into dogma. The logarithmic function, too, gains a new clarity here—not as an abstract operator but as a squeezing of dimensions and their building blocks.

Mathematics must therefore be rebuilt on rational, physically grounded principles. Constructs without logical or physical basis must be discarded, and numbers must be reclaimed as entirely real. The future of mathematics lies not in preserving inherited illusions, but in re-rooting the subject in coherence across mathematics, physics, and geometry.

DECLARATION STATEMENT

After aggregating input from all authors, I must verify the accuracy of the following information as the article's author.

- **Conflicts of Interest/ Competing Interests:** Based on my understanding, this article has no conflicts of interest.
- **Funding Support:** This article has not been funded by any organizations or agencies. This independence ensures that the research is conducted objectively and free from external influence.
- **Ethical Approval and Consent to Participate:** The content of this article does not necessitate ethical approval or consent to participate with supporting documentation.
- **Data Access Statement and Material Availability:** Data sharing does not apply to this article, as no datasets were generated or analysed during the study.
- **Author's Contributions:** Each author has individually contributed to the article. Dr Chinmoy Bhattacharya led the research, and Nishant Sahdev assisted him throughout the research.

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