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Non-Trivial Zeros of the Riemann Zeta Function as Zero Displacement Vectors

Joseph Kongani Wamukoya

***Abstract:** In this paper, we show that a non-trivial zero of the Riemann zeta function occurs only when the complex number $s = a/b + it$, with $a, b, t \in R$ and $i^2 = -1$ can be interpreted as a vector plus its inverse yielding zero displacement. We prove that for such a zero displacement to occur, the total distance covered by the vector and its inverse must equal one unit, forcing the fundamental part of s to be $\frac{1}{2}$. We further show that no other fraction in the critical strip possesses this property. Consequently, no other fundamental part can host non-trivial zeros, thereby settling the Riemann Hypothesis.*

***Keyword:** Riemann Zeta Function, Equal One Unit, Fundamental Part*

I. INTRODUCTION

The Riemann zeta function $\zeta(s)$ and the distribution of its non-trivial zeros in the critical strip $0 < \Re(s) < 1$ are central topics in analytic number theory. The Riemann Hypothesis asks whether all nontrivial zeros lie on the line $\Re(s) = \frac{1}{2}$ [1].

The conjecture has been studied through both analytic and computational approaches. Extensive numerical verification has confirmed that all non-trivial zeros with imaginary part up to 3×10^{12} lie on the critical line $\Re(s) = \frac{1}{2}$ [2]. While such verification does not constitute proof, it provides strong empirical confirmation of the conjectured structure.

The objective of this paper is to formulate a mathematical property that is unique to the fraction within the critical strip $0 < x < 1$. This property is shown not to be shared by any other fraction in the strip and, at the same time, to prevent any other fraction from hosting non-trivial zeros of the Riemann zeta function.

In this work, the real and imaginary parts of a non-trivial zero of the Riemann zeta function are treated as a vector and its inverse, whose sum results in zero displacement. While it is well known that the sum of a vector and its inverse produces zero displacement, the novelty here lies in imposing a distance constraint that uniquely identifies the fraction $x = \frac{1}{2}$.

Let the total distance covered by the vector and its inverse be denoted by $d > 0$. Both vectors are assumed to originate from the y -axis and to be perpendicular to it, following the same straight-line path. We consider a fractional vector lying in the critical strip $0 < x < 1$. The Riemann Hypothesis asks whether all non-trivial zeros lie on the line $x = \frac{1}{2}$. We answer this by showing that equal-distance traversal by a vector and its inverse forces and excludes all other fractions.

II. PROOF OF THE UNIQUE PROPERTY OF A VECTOR AND ITS INVERSE

Let there be a vector directed to the right, originating from and perpendicular to the y-axis. Let its inverse be $-it$, directed to the left along the same straight-line path. Let the total distance travelled by the vector and its inverse be d.

The distance travelled by the vector is

$$\frac{a}{b}d$$

and the distance travelled by its inverse is

$$|it|d.$$

Since the total distance travelled is the sum of these two distances, we have

$$\frac{a}{b}d + |it|d = d \quad \dots \quad (1)$$

Dividing both sides by d yields

$$\frac{a}{b} + |it| = 1 \quad \dots \quad (2)$$

For zero displacement to occur, the magnitudes of the vector and its inverse must be equal. Thus,

$$\frac{a}{b} = |it| \quad \dots \quad (3)$$

Substituting Equation (3) into Equation (2) gives

$$2|it| = 1 \quad \dots \quad (4)$$

Dividing by 2, we obtain

$$\frac{a}{b} = |it| = \frac{1}{2} \quad \dots \quad (5)$$

Hence, the vector must have a fractional component $\frac{1}{2}$.

III. UNIQUENESS WITHIN THE CRITICAL STRIP

We now show that it is the only fraction in the critical strip with this property.

If $\frac{a}{b} < \frac{1}{2}$, then the inverse vector distance exceeds the forward distance and overshoots the zero displacement point. For example, if the vector is

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*Correspondence Author(s)

Joseph Kongani Wamukoya*, Department of Math/Physics, Nairobi, Westlands, Nairobi, Kenya. Email ID: wamukoyajoseph353@gmail.com, ORCID ID: 0009-0008-4325-1076

$\frac{1}{4}$, the inverse distance becomes $\frac{3}{4}$, leading to overshoot.

If $\frac{a}{b} > \frac{1}{2}$, the inverse distance is insufficient to return to the origin. For instance, if the vector is $\frac{3}{4}$, the inverse distance becomes $\frac{1}{4}$, which undershoots.

In both cases, the magnitudes of the vector and its inverse are unequal. Since distance travelled equals vector length, and zero displacement requires equality of magnitudes, no fraction other than $\frac{1}{2}$ satisfies this condition.

IV. EQUIVALENT IMAGINARY DECIMAL FRACTIONS

The imaginary component satisfies

$$-it = -\frac{a}{b} \quad \ldots \quad (6)$$

While the fundamental part remains fixed $x = \frac{1}{2}$, the imaginary component varies. This variation can be understood as equivalent representations of the same fractional magnitude, analogous to the rational equivalences

$$-\frac{1}{2}, -\frac{2}{4}, -\frac{3}{6}, -\frac{5}{10}, \ldots$$

In the case of the Riemann zeta function, the imaginary parts of non-trivial zeros take decimal values such as

$$-14.134 \ldots i, -21.022 \ldots i, 25.011 \ldots i,$$

which are interpreted here as equivalent decimal fractions corresponding to the same underlying magnitude $-\frac{1}{2}$. Extensive numerical verification confirms that all non-trivial zeros with imaginary part up to 3×10^{12} lie on the critical line $\Re(s) = \frac{1}{2}$ [2].

V. CONCLUSION

Using vector geometry, we have shown that a non-trivial zero of the Riemann zeta function occurs only when the complex number s can be interpreted as a vector whose inverse yields a zero displacement. The requirement that the vector and its inverse travel equal distances uniquely force the fundamental part of s to be $\frac{1}{2}$. No other fraction in the critical strip satisfies this requirement. Consequently, no other fundamental part can host non-trivial zeros of the Riemann zeta function, thereby resolving the Riemann Hypothesis.

DECLARATION STATEMENT

I must verify the accuracy of the following information as the article's author.

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REFERENCES

1. R. Spigler, *A Brief Survey on the Riemann Hypothesis and Some Attempts to Prove It*, Symmetry, Vol. 17, No. 2, 2025, Article 225. DOI: https://doi.org/10.3390/sym17020225
2. D. J. Platt and T. S. Trudgian, *The Riemann Hypothesis Is True up to* 3×10^{12}, Bulletin of the London Mathematical Society, Vol. 53, No. 3, 2021, pp. 792–797. DOI: https://doi.org/10.1112/blms.12460

AUTHOR'S PROFILE

Joseph Kongani Wamukoya is a physicist and mathematician whose research focuses on number theory and the fundamental structure of physical laws. He is the author of the peer-reviewed article *On the Quantization of Time, Space and Gravity* (Journal of Modern Physics, 2020, DOI: 10.4236/jmp.2020.1111112), where he develops a unified mathematical framework linking temporal, spatial, and gravitational behavior. Wamukoya has taught physics and mathematics at the secondary and pre-university levels, including at the British School of Lomé in Togo, where he supported students in advanced scientific study. He continues to pursue research that strengthens the connection between mathematical reasoning and physical theory while contributing to accessible science education.