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A General Formula for the Probability of Winning in Sequential Turn-Based Games

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Abstract: ***This paper presents a general formula for calculating a player's probability of winning in a sequential, turn-based game with a constant success probability per trial. The problem extends the classical two-player probability models of dice tossing or coin flipping to an arbitrary number of n players. A compact proof based on the summation of a geometric series is provided, and examples demonstrate the correctness and applicability of the result. This formulation can serve as an educational tool for understanding probabilistic reasoning, sequences, and infinite series.***

Keywords: Probability, Successive Trials, Random Experiments, Sequential Games, Geometric Series, Binomial Distribution.

I. INTRODUCTION

Probability theory is one of the fundamental branches of mathematics that models uncertain events. A classic example is a game in which two or more players take turns tossing a coin or throwing a die until one succeeds. Traditional problems of this nature usually deal with two players, but the analysis can be generalized for any number of players n players.

In this paper, we derive a formula to find the probability of winning for each player in such a sequential game, assuming that:

- Each player has the same probability of success p on their turn.
- The probability of failure is $q = 1 - p$.
- The game continues until one player achieves success.

The formula provides a closed-form solution for the probability [1] of winning for each player, regardless of the total number of participants.

II. MAIN THEOREM AND PROOF

Theorem: If n players $A_1, A_2, \ldots, A_n$ take turns sequentially in a game where each has a probability p of success and $q = 1 - p$ of failure, then the probability that the k^{th} the player's win is given by:

$$P(A_k) = q^{k-1} \cdot \frac{p}{1-q^n}, \text{ for } k = 1,2,3,\ldots.,n.$$

Proof: Player A_1 can win in several possible ways:

- A_1 succeeds on the first round: probability p.
- All n players fail once, and then A_1 succeeds on her second chance: probability$q^n p$.
- All n players fail twice, and then A_1 succeeds on her third chance: probability $q^{2n}p$.

Therefore, the total probability that A_1 wins is:

$$P(A_1) = p + q^n p + q^{2n} p + q^{3n} p + \ldots..$$

This is an infinite geometric series with the first term $a = p$ and common ratio $r = q^n$.

For $|r| < 1$,

$$P(A_1) = \frac{a}{1-r} = \frac{p}{1-q^n}.$$

Similarly, each subsequent player's probability of winning is delayed by one additional failure factor q:

$$P(A_2) = q \cdot P(A_1),$$

$$P(A_3) = q^2 \cdot P(A_1),$$

$$P(A_n) = q^{n-1} \cdot P(A_1).$$

Hence,

$$P(A_k) = q^{k-1} \cdot \frac{p}{1-q^n}, \text{ for } k = 1,2,3,\ldots\ldots.,n.$$

Finally, since one of the n players must win,

$$\sum_{k=1}^{n} P(A_k) = P(A_1)(1 + q + q^2 + \ldots. + q^{n-1})$$

$$= \frac{p}{1-q^n} \cdot \frac{1-q^n}{1-q}$$

$$= \frac{p}{1-q} = 1.$$

Thus, the formula satisfies the total probability condition.

III. EXAMPLES AND APPLICATIONS

A. Example 1: Two Players (Dice Game)

Let A and B Throw a die alternately until one of them gets a six. Then $p = \frac{1}{6}$ and $q = \frac{5}{6}$.

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$$P(A) = \frac{p}{1-q^2} = \frac{\frac{1}{6}}{1-\left(\frac{5}{6}\right)^2} = \frac{6}{11},$$

$$P(B) = q \cdot P(A) = \frac{5}{6} \times \frac{6}{11} = \frac{5}{11}.$$

B. Example 2: Three Players (Coin Tossing)

Let A, B and C toss a coin alternately until one of them gets a head. If A starts,

Then $p = \frac{1}{2}$ and $q = \frac{1}{2}$.

$$P(A) = \frac{p}{1-q^3} = \frac{\frac{1}{2}}{1-\left(\frac{1}{2}\right)^3} = \frac{4}{7},$$

$$P(B) = q \cdot P(A) = \frac{1}{2} \times \frac{4}{7} = \frac{2}{7},$$

$$P(B) = q^2 \cdot P(A) = \left(\frac{1}{2}\right)^2 \times \frac{4}{7} = \frac{1}{7}.$$

Thus, their respective probabilities are $4:2:1$ for A, B and C.

IV. CONCLUSION

This paper presents a general and elegant solution for determining the probability of winning in sequential turn-based games. The result not only unifies several well-known problems in elementary probability but also provides an intuitive understanding of infinite geometric progressions in random experiments. The approach is simple enough to be introduced at the high-school level, yet general enough to inspire further exploration in stochastic modelling and game theory.

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DECLARATION STATEMENT

I must verify the accuracy of the following information as the article's author.

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AUTHOR'S PROFILE

Sakshya Vardhan Mishra is a senior secondary student (Class XII, PCM) at Sunbeam School, Mughalsarai, Dist: Chandauli, Uttar Pradesh, with a strong academic inclination towards mathematics, particularly probability theory. Through independent study and logical analysis, he derived a general formula for calculating the probability of winning in sequential, multi-player turn-based games, thereby extending classical probability models. His work demonstrates analytical clarity and early engagement with mathematical research. Sakshya is also an active cadet in the National Cadet Corps (NCC), where he has developed discipline, leadership, and perseverance that complement his academic pursuits. He intends to pursue higher studies in mathematics and related scientific disciplines.