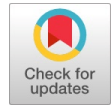


A Comparative Study on the Effects of Vertical and Horizontal Magnetic Fields on Double Diffusive Convection with Soret Effect in Couple-Stressed Nanofluids



Suman Sindhvani

Abstract: This study aims to conduct a comparative analysis of double diffusive convection in a Soret-induced couple-stressed nanofluid layer under the influence of uniform vertical and horizontal magnetic fields. The linear stability analysis is based on the standard mode technique. The Galerkin method has been applied to find the critical Rayleigh number and the corresponding wave number in terms of various parameters. The effects of the Soret parameter, magnetic field, Lewis number, modified diffusivity ratio, Concentration Rayleigh-Darcy number, and Solutal Rayleigh number on the system's stability have been investigated. It has been observed that Stationary convection remains unaffected by the relaxation parameter, and the critical wave number is a function of both the couple stress parameter and the magnetic field; however, stationary convection is stabilised by the couple stress. In double diffusive convection under a magnetic field, the Darcy number also comes into play and has been observed to provide a stabilising effect on stationary convection. In comparison to ordinary fluids, convection sets up earlier in nanofluids. A comparative graphical analysis has been conducted to depict the effects of couple stress and Soret parameter on the stability of a nanofluid in the presence of both vertical and horizontal magnetic fields. It has been observed that the magnetic field still has a stabilising effect in both cases. However, in the case of a vertical magnetic field, the critical value of the stationary Rayleigh number is obtained at a significantly larger wave number compared to that in the case of a horizontal magnetic field, with all parameters remaining the same. The literature survey indicates that no study has investigated the comparison of the effect of vertical and horizontal magnetic fields on double diffusive convection in a couple stress nanofluid layer with a Soret factor. The present study examines the comparative analysis of the effect of vertical and horizontal magnetic fields on Soret-induced double diffusive convection in a couple-stress nanofluid horizontal layer.

Keywords: Magnetic Field, Rayleigh Number, Couple-Stressed Nanofluid, Soret Factor

I. INTRODUCTION

Creating a cutting-edge cooling system to cool crystal silicon mirrors used in high-intensity X-ray sources is one of the earliest applications of nanofluid research.

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Because metallic nanoparticles enhance the effective thermal conductivity of coolants and microchannels increase the effective heat transfer area, this technique may offer more effective cooling through the use of microchannels filled with nanofluids.

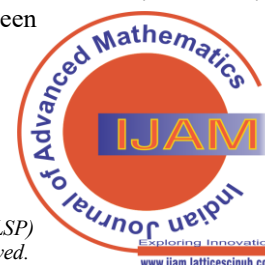
In MHD power generators, electronic device cooling systems and metal casting, the impact of a magnetic field on the movement of electrically conducting fluids through a vertical plate is crucial. To enhance the heat transfer performance of such devices, the working medium is a nanofluid with enhanced thermal conductivity. A magnetic field can be used to control the flow and temperature fields after an electrically conducting nanofluid has been poured into the area between the plates. When a magnetic field is applied, a Lorentz force is created, which interacts with buoyancy. Yadav investigated the effect of magnetic fields on the onset of nanofluid convection, assuming that the boundaries were imposed by temperature. In unsteady nanofluid flow, the magnetic field effect was studied by Sheikholeslami et.al. [4]. An analytical investigation was conducted for various governing parameters.

A macroscopic filtration model for natural convection in a Darcy-Maxwell nanofluid-saturated porous layer with no nanoparticle flux at the boundary was studied by Jaimala et al. [1]. Shankar et al. [3] investigated the effect of magnetic fields in couple stress fluids. They discovered that a magnetic field slows down the onset of instability, but as the couple stress parameter increases, the opposite behaviour is observed. In the thermo-solutal convection issue for a couple of stresses, Kumar [2] found that both the magnetic field and the couple stress had both stabilising and destabilising effects. Sithole et. al. [6] investigated the issue of thermal radiation and heat generation in a pair stress nanofluid in the presence of a magnetic field.

The author studied the linear stability graphical analysis for this case [5] in the presence of a vertical magnetic field. The present study examines the comparison of vertical and horizontal magnetic fields on Soret-induced double diffusive convection in a couple-stress nanofluid horizontal layer.

II. MATHEMATICAL FORMULATION

An infinite isotropic couple-stressed nanofluid porous layer of width a between two horizontal planes, with temperatures at the lower and upper boundaries are T_l^* and T_r^* (T_l^* being greater than T_r^*) has been considered. A uniform horizontal magnetic field



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$M^* = (M_0^*, 0, 0)$ acts on the system. The governing equations are as follows:

$$\nabla^* \cdot q_d^* = 0 \quad \dots (1)$$

$$\frac{1}{K}(\mu - \mu_c \nabla^{*2})q_d^* = \left(1 + \lambda^* \frac{\partial}{\partial t^*}\right) \left[\{-\nabla^* p^* + (\psi^* \rho_p + (1 - \psi^*)\{\rho(1 - \beta_t(T^* - T_r^*) - \beta_c(S^* - S_r^*))\})g\} + \frac{\mu_c}{4\pi}(\nabla^* \times M^*) \times M^* \right] \quad \dots (2)$$

$$(\rho c)_M \frac{\partial T^*}{\partial t^*} + (\rho c)_F q_d^* \cdot \nabla^* T^* = k_m \nabla^{*2} T^* + \in (\rho c)_P \left[B_d \nabla^* \psi^* \cdot \nabla^* T^* + \left(\frac{B_t}{T_r^*}\right) \nabla^* T^* \cdot \nabla^* T^* \right] \quad \dots (3)$$

$$\frac{\partial S^*}{\partial t^*} + \frac{1}{\epsilon} q_d^* \cdot \nabla^* S^* = S_d \nabla^{*2} S^* + S_{ct} \nabla^{*2} T^* \quad \dots (4)$$

$$\frac{\partial \psi^*}{\partial t^*} + \frac{1}{\epsilon} q_d^* \cdot \nabla^* \psi^* = B_d \nabla^{*2} \psi^* + \frac{B_t}{T_r^*} \nabla^{*2} T^* \quad \dots (5)$$

$$\left(\frac{\partial}{\partial t^*} + \frac{1}{\epsilon} (q_d^* \cdot \nabla^*)\right) M^* = (M^* \cdot \nabla^*) \frac{1}{\epsilon} q_d^* + \eta \nabla^{*2} M^* \quad \dots (6)$$

where $\nabla^* \cdot M^* = 0$, $\eta = \frac{1}{4\pi\mu_e\sigma}$ and

$q_d^* = (u_{1d}^*, u_{2d}^*, u_{3d}^*)$ along with boundary conditions

$$q_d^* = 0, T^* = T_l^*, S^* = S_l^*, B_d \frac{\partial \psi^*}{\partial z^*} + \frac{B_t}{T_r^*} \frac{\partial T^*}{\partial z^*} = 0 \text{ at } z^* = 0 \quad \dots (7)$$

$$q_d^* = 0, T^* = T_r^*, S^* = S_r^*, B_d \frac{\partial \psi^*}{\partial z^*} + \frac{B_t}{T_r^*} \frac{\partial T^*}{\partial z^*} = 0 \text{ at } z^* = a \quad \dots (8)$$

Introducing non-dimensional parameters in taking ψ_0^* as a reference scale for volumetric fraction of nanoparticles, $\alpha_m \left(= \frac{k_m}{(\rho c)_F} \right)$ as the thermal diffusivity of the porous medium and $\sigma \left(= \frac{(\rho c)_M}{(\rho c)_F} \right)$ as the heat capacity ratio parameter,

$$(X, Y, Z) = \frac{(x^*, y^*, z^*)}{a}, \quad t = \frac{t^* \alpha_m}{\sigma a^2}, \quad (u_{1d}, u_{2d}, u_{3d}) = \frac{(u_{1d}^*, u_{2d}^*, u_{3d}^*) a}{\alpha_m}, \quad p = \frac{p^* K}{\mu \alpha_m}, \quad \psi = \frac{\psi^* - \psi_0^*}{\psi_0^*}, \quad T = \frac{T^* - T_r^*}{T_l^* - T_r^*}, \quad S = \frac{S^* - S_r^*}{S_l^* - S_r^*}, \quad \lambda = \frac{\lambda^* \alpha_m}{a^2}, \quad (M_X, M_Y, M_Z) = \frac{(M_X^*, M_Y^*, M_Z^*)}{M_0^*}.$$

Non-dimensional form of equations is given by

$$\nabla \cdot q = 0 \quad \dots (9)$$

$$(q - \mathbb{C} \nabla^2 q) = \left(1 + \frac{\lambda}{\sigma} \frac{\partial}{\partial t}\right) \left[\left(-\nabla p - R_m \hat{e}_z - R_n \psi \hat{e}_z + R_a T \hat{e}_z + \frac{R_s}{Ln} S \hat{e}_z\right) + \frac{P_1}{P_{1M}} Q D_a (\nabla \times M) \times M \right] \quad \dots (10)$$

$$\frac{\partial T}{\partial t} + (q \cdot \nabla) T = \nabla^2 T + \frac{N_b}{Le} \nabla \psi \cdot \nabla T + \frac{N_a N_b}{Le} \nabla T \cdot \nabla T \quad \dots (11)$$

$$\frac{1}{\sigma} \frac{\partial S}{\partial t} + \frac{1}{\epsilon} q \cdot \nabla S = \frac{1}{Ln} \nabla^2 S + N_{ct} \nabla^2 T \quad \dots (12)$$

$$\frac{1}{\sigma} \frac{\partial \psi}{\partial t} + \frac{1}{\epsilon} (q \cdot \nabla) \psi = \frac{1}{Le} \nabla^2 \psi + \frac{N_a}{Le} \nabla^2 T \quad \dots (13)$$

$$\frac{1}{\sigma} \frac{\partial M}{\partial t} + \frac{1}{\epsilon} (q \cdot \nabla) M = \frac{1}{\epsilon} (M \cdot \nabla) q + \frac{P_1}{P_{1M}} \nabla^2 M \quad \dots (14)$$

Here $R_a \left(= \frac{\rho g \beta K a (T_l^* - T_u^*)}{\mu \alpha_m} \right)$, $R_n \left(= \frac{(\rho_p - \rho) \psi_0^* g K a}{\mu \alpha_m} \right)$, $R_m \left(= \frac{\rho_p \psi_0^* + \rho(1 - \psi_0^*) g K a}{\mu \alpha_m} \right)$, $R_s \left(= \frac{\rho \beta_c g a K (S_l^* - S_u^*)}{\mu S_d} \right)$ They are thermal, concentration, basic density, and solutal Rayleigh-Darcy numbers, respectively. $\mathbb{C} \left(= \frac{\mu_c}{\mu a^2} \right)$ is a couple of stress parameters, $P_1 \left(= \frac{\mu}{\rho \alpha_m} \right)$ and $P_{1M} \left(= \frac{\mu}{\rho \eta} \right)$ are Prandtl numbers, $Q \left(= \frac{\mu_e M_0^2 a^2}{4\pi \mu \eta} \right)$ is the Magnetic Chandrasekhar number, $D_a \left(= \frac{K}{a^2} \right)$ is the Darcy number, $N_{ct} \left(= \frac{S_{ct} (T_l^* - T_u^*)}{\alpha_m (S_l^* - S_u^*)} \right)$ is Soret parameter, $N_a \left(= \frac{B_t (T_l^* - T_r^*)}{B_d T_r^* Q_0^*} \right)$ and $N_b \left(= \frac{(\rho c)_P \epsilon Q_0^*}{(\rho c)_F} \right)$ The modified diffusivity ratio and modified particle density increment, respectively. $Le \left(= \frac{\alpha_m}{B_d} \right)$ and $Ln \left(= \frac{\alpha_m}{S_d} \right)$ are Lewis numbers for the nanofluid and the salt, respectively?

A. Basic State and Perturbed State

The time-independent basic state of the nanofluid is described as

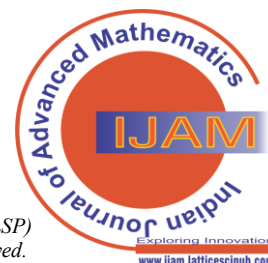
$$q = 0, p = p_{bs}(Z), T = T_{bs}(Z), \psi = \psi_{bs}(Z), S = S_{bs}(Z), M = \hat{e}_X \quad \dots (15)$$

where the suffix “bs” refers to the basic flow. Following Chandrasekhar [3], the basic volume fraction and temperature of nanoparticles are given as $T_{bs} = 1 - Z$, $\psi_{bs} = \psi_0 + N_a Z$, and $S_{bs} = 1 - Z$. In the basic state, we superimpose perturbations in the form. $q = q'$, $p = p_{bs} + p'$, $S = S_{bs} + S'$, $\psi = \psi_{bs} + \psi'$, $M = \hat{e}_X + M'$

Linearised perturbation equations of Couple Stress nanofluid are obtained as $\left(\frac{1}{\sigma} \frac{\partial}{\partial t} - \frac{P_1}{P_{1M}} \nabla^2\right) \left[(\nabla^2 - \mathbb{C} \nabla^4) u'_{3d} - \left(1 + \frac{\lambda}{\sigma} \frac{\partial}{\partial t}\right) \left(R_a \nabla_H^2 T' - R_n \nabla_H^2 \psi' + \frac{R_s}{Ln} \nabla_H^2 S' \right) \right]$

$$= \left(1 + \frac{\lambda}{\sigma} \frac{\partial}{\partial t}\right) Q \frac{P_1}{P_{1M}} \frac{D_a}{\epsilon} \nabla^2 \frac{\partial^2 u'_{3d}}{\partial X^2} \quad \dots (16)$$

$$\frac{\partial T'}{\partial t} - u'_{3d} = \nabla^2 T' - \frac{N_a N_b}{Le} \frac{\partial T'}{\partial Z} - \frac{N_b}{Le} \frac{\partial \psi'}{\partial Z} \quad \dots (17)$$



$$\frac{1}{\sigma} \frac{\partial S'}{\partial t} - \frac{u'_{3d}}{\epsilon} = \frac{1}{Ln} \nabla^2 S' + N_{ct} \nabla^2 T' \dots (18)$$

$$\frac{1}{\sigma} \frac{\partial \psi'}{\partial t} + \frac{1}{\epsilon} N_a u'_{3d} = \frac{1}{Le} \nabla^2 \psi' + \frac{N_a}{Le} \nabla^2 T' \dots (19)$$

with the boundary conditions

$$u'_{3d} = 0, T' = 0, S' = 0, \frac{\partial \psi'}{\partial Z} + N_a \frac{\partial T'}{\partial Z} = 0 \text{ at } Z = 0 \text{ and } Z = 1 \dots (20)$$

B. Linear Stability Analysis

$$R_a = \frac{\sigma}{\epsilon \alpha^2} \left[\frac{R_s \alpha^2 (\lambda s + \sigma) (\sigma A \delta^2 + s) (\sigma \delta^2 + s L e) (\delta^2 + s) \{ \delta^2 (\epsilon N_{ct} - 1) - s \} - R_n N_a \alpha^2 (\lambda s + \sigma) (\delta^2 \sigma + s L n) (A \delta^2 \sigma + s) \{ \delta^2 (\epsilon + L e) + s L e \} + \epsilon (\delta^2 \sigma + s L n) (\delta^2 \sigma + s L e) (\delta^2 + s) \{ A \sigma \delta^4 + B l^2 \delta^2 (\sigma + \lambda \delta) + s \delta^2 + s C \delta^4 + A C \sigma \delta^6 \}}{(\sigma \delta^2 + s L n) (\sigma \delta^2 + s L e) (\lambda s + \sigma) (A \delta^2 \sigma + s)} \right] \dots (22)$$

where $\delta^2 = \pi^2 + \alpha^2$. $\alpha = (L^2 + M^2)^{1/2}$ Equation (22), on taking $s=0$, results in the following Rayleigh number

$$R_a^{st} = \frac{\delta^4}{\alpha^2} - \left(1 + \frac{L e}{\epsilon} \right) R_n N_a + \frac{Q D_a L^2 \delta^2}{\epsilon \alpha^2} - \frac{R_s}{\epsilon} (1 - \epsilon N_{ct}) + \frac{C \delta^6}{\alpha^2} \dots (23)$$

To obtain the critical value of the Rayleigh number

$\frac{d R_a^{st}}{d \alpha} = 0$ And the following critical wave number equation is obtained.

$$2 C (\alpha^2)^3 + (3 \pi^2 C + 1) (\alpha^2)^2 - (\pi^4 + C \pi^6 + \frac{Q D_a L^2}{\epsilon} \pi^2) = 0 \dots (24)$$

Equations (23) and (24), which give the stationary Rayleigh number and critical wave equation, are similar to the

Following the linear stability theory by Chandrasekhar [3], the perturbations are taken in the form

$$(\psi', T', u'_{3d}, S') = [\Phi(Z), \Theta(Z), \Omega(Z), \Psi(Z)] e^{st + i L X + i M Y} \dots (21)$$

where L and M are dimensionless wave numbers in X and Y Directions respectively. Application of the Galerkin-type weighted residuals method with first approximation ($N=1$) gives $\Omega = A_1 \sin \pi Z$, $\Theta = B_1 \sin \pi Z$, $\Phi = -N_a C_1 \sin \pi Z$, $\Psi = D_1 \sin \pi Z$ and leads to the following Rayleigh number

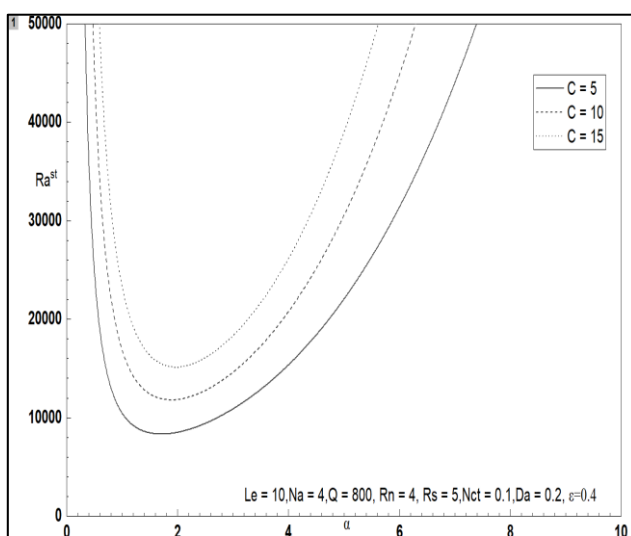
following equations were obtained by the author [5] in the case of a vertical magnetic field.

$$R_a^{st} = \frac{\delta^4}{\alpha^2} - \left(1 + \frac{L e}{\epsilon} \right) R_n N_a + \frac{Q D_a \pi^2 \delta^2}{\epsilon \alpha^2} - \frac{R_s}{\epsilon} (1 - \epsilon N_{ct}) + \frac{C \delta^6}{\alpha^2} \dots (25)$$

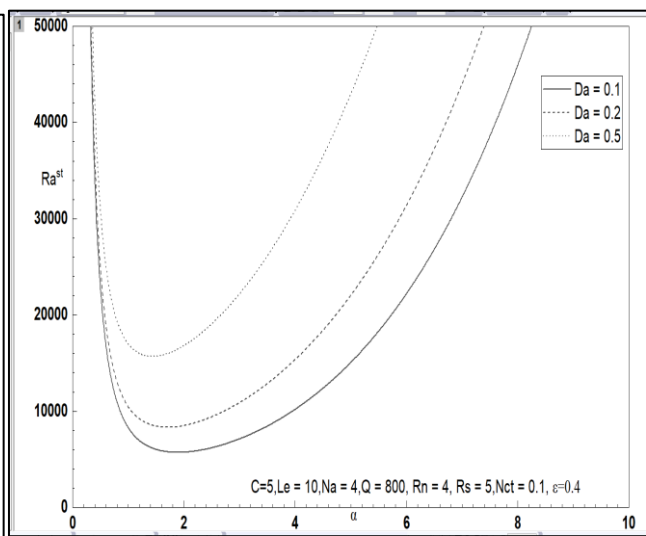
$$2 C (\alpha^2)^3 + (3 \pi^2 C + 1) (\alpha^2)^2 - (\pi^4 + C \pi^6 + \frac{Q D_a}{\epsilon} \pi^4) = 0 \dots (26)$$

III. RESULTS AND DISCUSSION

The stationary convection curves for the Rayleigh number R_a versus the wave number $\alpha (= L)$ are in Fig 1(a)-(f), where $C = 5, N_a = 4, D_a = 0.2, Le = 10, R_n = 4, \epsilon = 0.4, Q = 800, R_s = 5, N_{ct} = 0.1$.

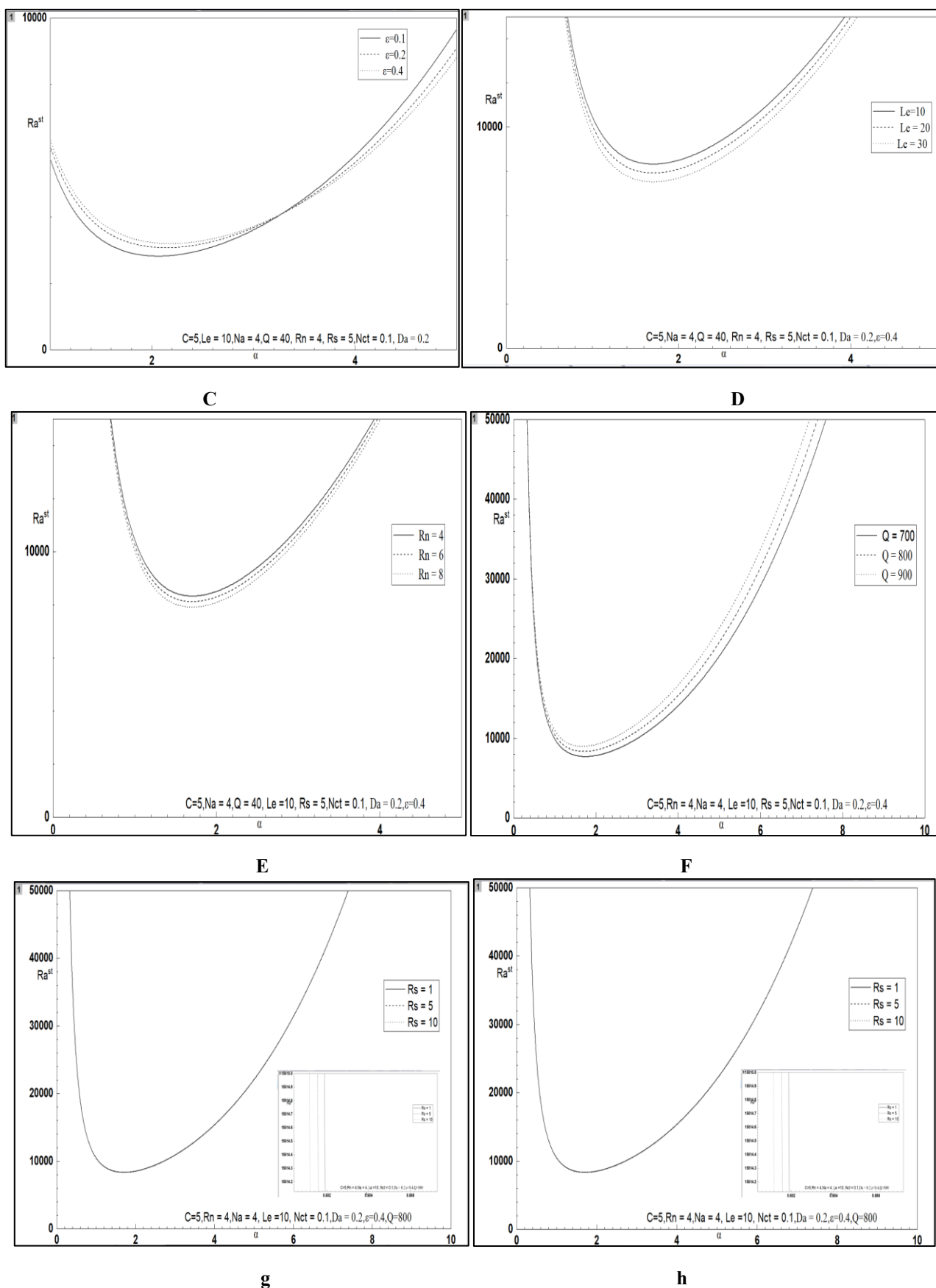


A



B

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[Fig.1: Linear Stationary Convection with Wave Number in the Presence of a Horizontal Magnetic Field for Different Values of (a) C (b) Da (c) ε (d) Le (e) R_n (f) Q (g) R_s (h) N_{ct}]

IV. CONCLUSION

In this analysis, the following results have been obtained:

- Stationary convection remains unaffected by the relaxation parameter.
- Critical wave number is a function of the couple stress parameter as well as the magnetic field.
- Stationary convection gets stabilised by the couple stress.
- As compared to ordinary fluids, convection sets up earlier in nanofluid.
- Magnetic field stabilises the nanofluid layer.
- Porosity has a stabilising as well as a destabilising effect on convection.
- Soret parameter promotes the stability of the flow.
- Behaviour of solutal Rayleigh number is destabilising, thus resulting in early convection.

Furthermore, a comparison has been made to observe the stability under both vertical and horizontal magnetic fields. It is interesting to observe that the magnetic field still has a stabilising effect in both cases. However, in the case of a vertical magnetic field, the critical value of the stationary Rayleigh number is obtained at a significantly larger wave number compared to the case of a horizontal magnetic field, with all parameters remaining the same.

DECLARATION STATEMENT

I must verify the accuracy of the following information as the article's author.

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- **Funding Support:** This article has not been funded by any organizations or agencies. This independence ensures that the research is conducted with objectivity and without any external influence.
- **Ethical Approval and Consent to Participate:** The content of this article does not necessitate ethical approval or consent to participate with supporting documentation.
- **Data Access Statement and Material Availability:** The adequate resources of this article are publicly accessible.
- **Author's Contributions:** The authorship of this article is contributed solely.

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