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An Innovative Approach to Find Remainder

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***Abstract:** Let r be the remainder when x! is divided by p, $(x, p) \in N$, $x < p$. Then the value of r is given by the minimum value of k for which $(-1)^{(m-1)} (m-1)!\, k + 1 = 0 \pmod{p}$, where $m = p - x$, $k \in N$.*

***Index Terms:** Remainder, Number Theorem, Factorial Remainder, Remainder using Wilson's Theorem*

I. INTRODUCTION

Theorem: [Wilson] *According to Wilson's theorem [1] $(p-1)! + 1 = 0 \pmod{p}$, p is prime, i.e., when $(p-1)! +1$ is divided by p, the remainder is 0 [2]*

or $(p-1)! + 1 = 0 \pmod{p}$, or $(p-1)! = -1 \pmod{p}$ …………………………… (A)

Instead of $(p-1)!$ If we have x! where $x < p$ and $x \in N$, then how do we obtain the remainder? There are complicated modulo methods by which we can ultimately obtain the remainder, but in this paper, I present an innovative technique and theorem to calculate the remainder straightforwardly.

PURPOSE: The primary objective of my work is to develop a concise formula for finding the remainder in a simplified manner.

II. METHODOLOGY

Theorem: *Let r be the remainder when x! is divided by p, $(x, p) \in N$, $x < p$. Then the value of r is given by the minimum value of k for which*

$$(-1)^{(m-1)} (m-1)!\, k + 1 = 0 \pmod{p}, \text{ where } m = p - x,\ k \in N.$$

Proof: *According to Wilson's theorem [1]: for any prime p, $(p-1)! + 1$ is divisible by p,*

or $(p-1)! + 1 = 0 \pmod{p}$, or $(p-1)! = -1 \pmod{p}$ [2]

Now the question is, in place of $(p-1)!$ if we are given $(p-2)!$ or in general $(p-m)!$ $m <= p$,

then how to get r, from $x! = r \pmod{p}$ where $x = (p-m)$

Let $x=(p-2)$, we can write,

$(p-2)! = k \pmod{p}$

or $(p-2)!\,(p-1) = k(p-1) \pmod{p}$

or $(p-1)! = (-1)\, k \pmod{p}$

Or $(p-1)! +1= [(-1) k+1] \pmod{p}$,

[by (A)], $[(-1) k+1] =0 \pmod{p}$ and k is the minimum value for which $[(-1) k+1]$ is divisible by p.

If $x = (p-3)!$ then we have to find k for which $(p-3)! = k \pmod{p}$

or $(p-3)!\,(p-2)\,(p-1) = k(p-1)\,(p-2) \pmod{p}$

or $(p-1)! = (-1)\,(-2)\, k \pmod{p} = [(-1)^2\, 2!\, k + 1] \pmod{p}$ and by (A) $[(-1)^2\, 2!\, k]$ should be divisible for minimum value of k.

Thus, if $x =(p-m)$, we can write

$(p-m)! = k \pmod{p}$ or,

$(p-m)!\,(p-m+1)\,(p-m+2) \ldots\ldots\ (p-2)\,(p-1) = [(-1)\,(-2)\,(-3) \ldots\ldots\ \{-(m-1)]\, k \pmod{p}$

By (A), $[(-1)\,(-2)\,(-3) \ldots\ldots\ \{-(m-1)]\, k + 1$ is divisible by p.

The remainder r is given by the minimum value of k for which $[(-1)\,(-2)\,(-3) \ldots\ldots\ \{-(m-1)\}\, k+1]$ is divisible by p.

In other words, the remainder is given by the minimum value of k for which

$[(-1)^{(m-1)} (m-1)!\, k + 1] =0 \pmod{p}$

III. VALIDATION OF THE THEOREM

Example 1. *Remainder when 4! is divided by 7, or $4! = r \pmod{7}$.*

Remainder r is given by the minimum value of k, $k \in N$, for which $(-1)\,(-2)\, k+1$ is divisible by 7, and the minimum such value of k is 3. Hence, the remainder is 3.

Example 2. *Remainder when 9! is divided by 13, or $9! = r \pmod{13}$.*

Remainder r is given by the minimum value of k, $k \in N$, for which $(-1)\,(-2)\,(-3)\, k+1$ is divisible by 13, and the minimum such value of k is 11. Hence, the remainder is 11.

***Example 3**: $12! = r \pmod{17}$,* then to find r, we have to calculate the minimum value of k for which $(-1)^4\, 4!\, k + 1$ is divisible by 17

Or $24k+1$ is divisible by 17. Or $7k+1$ is divisible by 17. The minimum value of k is 12. So, the remainder is $k=12$.

Verification of result: $12! = 479001600/17 = 12$.

***Example 4**: $97! = r! \pmod{101}$. Then, to find r, we have to calculate the minimum value of k for which $(- 1)$ ^33!* $k + 1$ is divisible by 101 Or $-6k+1$ is divisible by 101. So, the remainder is $k=17$.

IV. CONCLUSION AND FUTURE WORK

In this paper, I have presented an innovative and efficient approach to determine the remainder when the factorial of a positive integer n! is divided by a prime number p. The proposed method simplifies the computational complexity associated with large factorials, especially for cases where direct computation is infeasible due to factorial growth. By leveraging properties of modular arithmetic, Wilson's Theorem, and pattern recognition in modular sequences, this approach enhances the accuracy and efficiency of remainder calculation, which has significant applications in number theory, cryptography, and competitive mathematics.

While the current method demonstrates promising results, several avenues for

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future exploration remain. I plan to generalize the approach to handle composite moduli, extending its utility beyond prime divisors. Additionally, incorporating optimization techniques for tremendous n values can make this method applicable to high-performance computing and cryptographic algorithms. Future work may also focus on developing closed-form expressions or recurrence relations for specific classes of problems, as well as integrating this method with machine learning models to predict modular patterns.

Further research into the algebraic structure and periodicity of factorial remainders, along with computational experiments, will contribute to strengthening the theoretical foundation of this approach and expanding its practical applications.

Originality: The formula I have developed is unique in its domain of calculation, as indicated by my literature survey and Turnitin Check.

DECLARATION STATEMENT

I must verify the accuracy of the following information as the article's author.

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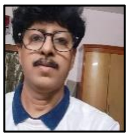
AUTHOR'S PROFILE



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