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Impacts of Some Definitions on Algebra of Differential Operators for Noncommutative Algebras

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***Abstract:** Rings of differential operators are one of the most important noncommutative (associative) algebras. They play an important role in the representation theory of Lie algebras and the algebraic analysis of systems of partial differential equations. However, If A is a commutative and unitary algebra on a field k, Grothendieck defined the ring of differential operators on the algebra A, denoted by D(A), as follows:*

$D(A):=\cup D^n(A)$,

where $D^{-1}(A)=0$ and for $n\in\mathbb{N}$, (1)

$D^n(A):=\{u\in End_k(A):[u, a]=ua-au \in D^{n-1}(A), \forall a\in A\}$.

In this paper, we show that with this definition, the algebra of differential operators is no longer rich when it is a noncommutative algebra.

***Keywords:** Differential Operators, Weyl Algebra, Noncommutative Algebra, Derivation on an Algebra.*

I. INTRODUCTION

In this paper, k is a zero characteristic field. The concept of differential operators on algebras was initially presented as (1) in [7] by Grothendieck. This approach has permitted to study of differential operators on commutative algebras. Indeed from this definition, it is well known that in characteristic zero, the algebra of differential operators on the polynomial algebra in n variables is the n-th noncommutative algebra that has almost the same properties as Weyl algebra. Stafford uses it to describe in terms of differential operators in [10], the endomorphisms of a right ideal of a Weyl algebra. Also, This approach allowed Bouchaïb El Boufi to extend in [4], a result of I.M. MUSSON on finite type algebras, to finite type A-modules without torsion.

However, the discussion that took place on a recursive description of algebras of differential operators on diagrams, and therefore on commutative algebras in terms of commutators (see in [7], p.42-43), interested Hazewinkel to know whether something more or less similar can be done for noncommutative algebras. This would be of particular interest for homotopy algebras, higher derivative algebras (see in [1] and [2]) and for the theory of deformation of algebras and diagrams to Gerstenhaber-Shack (see in [3] and [6]). For that, Michiel Hazewinkel proposed in [8], a new definition of the ring of differential operators [12].

In this paper, we show that Hazewinkel's approach is the best for studying the algebra of differential operators on a noncommutative algebra [13].

This paper is organized as follows. In section 2, we give some definitions, notations, and properties of differential operator algebra. In section 3, algebra is no longer commutative [14]. Firstly, focusing on Weyl algebras, we show that the algebra of differential operators loses under the first approach some of these module properties, and does not necessarily contain all the multiplicative morphisms and derivations. Secondly, we show that with Hazewinkel's definition, the algebra of differential operators on a commutative case [15].

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II. PRELIMINARIES

In this section, we present some definitions, notations, basic properties, and theorems, which are necessary for the best understanding of this paper.

A. Notation

1. $L_A:=\{l_a\in End_k(A),\ a\in A\}$, where $l_a(x)=ax$, for all $x\in A$.
2. $R_A:=\{r_a\in End_k(A),\ a\in A\}$, where $r_a(x)=xa$, for all $x\in A$
3. $Der_k(A)$ denotes the space of derivatives on a k-algebra A.
4. $A_n(k)$ denotes the n-th Weyl algebra.
5. For all $g\in A_n(k)=k< x_1,..,x_n,y_1,..,y_n>$, $g_{xq}'=((\partial g)/(\partial_{xq}))$ and $g_{yq}'=((\partial g)/(\partial_{yq}))$.

The first approach of differential operator, which has been defined for commutative algebra, is:

i. Definition

The ring of differential operators on a commutative and unitary k-algebra A is defined by:

$D(A):=\cup D^n(A)$,

where $D^{-1}(A)=0$ and for $n\in\mathbb{N}$,

$D^n(A):=\{u\in End_k(A):[u, a]=ua-au \in D^{n-1}(A), \forall a\in A\}$.

With ua and au are elements of $End_k(A)$ defined by:

$\forall x\in A$, $ua(x)=u(ax)$ and $au(x)=a(u(x))$.

Any element $u\in D^n(A)$ is called a differential operator of order n on A.

With this approach, some important properties of the ring of differential operators on commutative and unitary k-algebra are:

ii. Proposition

([11])

1. $End_k(A)$ is a $(D(A),D(A))$-bimodule.

2. let $n \geq 0$ be an integer, $D^n(A)$ is a (A,A) –sub-bimodule of $End_k(A)$.
3. D(A) is a (A,A)-sub-bimodule of $End_k(A)$.
4. D(A) is a subalgebra of .$End_k(A)$.

B. Proposition

([11]). Let $m,n \in \mathbb{N}$.

1. $D^0(A) = End_A(A)$
2. $D^n(A) \subseteq D^{n+1}(A))$.
3. $D^m(A).D^n(A) \subseteq D^{m+n}(A)$.
4. $[D^m(A).,D^n(A)] \subseteq D^{m+n-1}(A)$.
5. D(A) is a subalgebra of $End_k(A)$.

i. Proposition

([9]) For all k-algebra A, $D^1(A) = A \oplus Derk(A)$.

ii. Proposition

([9]). D(A) is an algebra filtered by the increasing sequence of submodules $(D^n(A))_{n\in\mathbb{N}}$.

iii. Definition

([6]). The n-th Weyl algebra is associative, unitary and noncommutative algebra, generated by 2n elements $x_1,\dots,x_n$ and $y_1,\dots,y_n$, which satisfy the following defined relations:

$$1. [x_i, x_j] = [y_i, y_j] = 0.$$

$$2. [y_i, x_j] = \delta_{ij}, for\ all\ i, j = 1, \dots, n$$

iv. Definition

Let A be a ring.

A derivation δ on A is interior if there exists $c \in A$ such that: $\delta(x) = [c, x] = cx - xc$, for all $x \in A$.

v. Proposition

(Seen in [5])

All non-zero derivations on Weyl algebra are interior.

Hazewinkel defines the ring of differential operators as follows:

C. Definition

([8]). The ring of differential operators on the algebra A is defined as follows:

$D(A) = \cup D_n(A)$,

where $D_{-1}(A) = 0$ and for $n \in \mathbb{N}$,

$D_n'(A) = \{u \in End_k(A) : [u, a] = ua - au \in D_{n-1}(A), \forall a \in A\}$

$D_n(A) = L_A\ D_n'(A) L_A$

Any element $u \in D_n(A)$ is called a differential operator of order n on A.

III. DIFFERENTIAL OPERATORS ON A NONCOMMUTATIVE ALGEBRA

In this section, A is a noncommutative unitary algebra.

We show that the Hazewinkel approach is the best for studying the algebra of differential operators on a noncommutative algebra.

A. Algebra of Differential Operators on a Noncommutative Algebra According to the First Approach

Compared to the commutative case, $D^n(A)$ and D(A) change structure algebraic when the algebra is non-commutative, as the proposition announces the following.

i. Proposition

Let $n \in \mathbb{N}$.

1. $D^0(A) = R_A$.
2. $D^n(A)$ is a subgroup of $End_k(A)$.
3. $R_A .D^n(A) \subseteq D^n(A)$ and $D^n(A). R_A \subseteq D^n(A)$.
4. $D^n(A)$ is a $(R_A - R_A)$ –sub-bimodule of $End_k(A)$.
5. D(A) is a $(R_A - R_A)$ –sub-bimodule of $End_k(A)$.

Proof: Let $n \in \mathbb{N}$.

1. For all $u \in D^0(A)$, we have $u = r_{u(1)}$. Thus, $D^0(A) \subset R_A$. Since $R_A \subset D^0(A)$, so $D^0(A) = R_A$
2. We know that $0_{Endk(A)} \in D^n(A)$ and $D^n(A) \subseteq End_k(A)$.

Then, we show by induction on n that $D^n(A)$ is sable.

- For n=0, $D^0(A) = R_A$. Then, $D^0(A)$ is sable.
- Now, assume that the result is true for an integer $n \geq 0$ and prove it for n+1.

Let $u,v \in D^{n+1}(A)$ and $a \in A$. Since $[u, a], [v, a] \in D^n(A)$, thus $[u\text{-}v, a] = [u, a] - [v, a] \in D^n(A)$.

Hence, $u\text{-}v \in D^{n+1}(A)$. It follows that $D^n(A)$ is a subgroup of $End_k(A)$.

3. By induction on $n \in \mathbb{N}$.

- For n=0, $D^0(A) = R_A$. Then, $R_A.D^0(A) \subseteq D^0(A)$.
- Now, assume that the result is true for an integer $n \geq 0$ and prove it for n+1.

Let $a \in A$ and $u \in D^{n+1}(A)$. We have

$[r_a \circ u, b] = r_a \circ [u, b] \in D^n(A)$, for all $b \in A$.

Then, $r_a \circ u \in D^{n+1}(A)$. Hence, $R_A. D^n(A) \subseteq D^n(A)$,for all $n \in \mathbb{N}$.

Similarly, we show that $D^n(A). R_A \subseteq D^n(A)$, for all $n \in \mathbb{N}$.

4. From 2 and 3, we obtain this property.
5. It is the consequence of the previous one. □ .

Besides, on Weyl algebras, we show that the algebra of differential operators loses under the first approach some properties.

In this paragraph, we study the properties of the algebra of differential operators on a Weyl algebra.

ii. Remark

Let $g \in A_n(k) = k<x_1,..,x_n, y_1,...y_n>$. We have:

1. $[g, x_m] = g_{ym}'$, for all $m \in 1,n$.
2. $[g, y_m] = - g_{xm}'$, for all $m \in 1,n$.

iii. Lemma

Let $n \in \mathbb{N}$, $P \in k[x_1,., x_n]$ and $Q \in k[y_1., y_n]$. We have:

1. $[P, x_m y_m] \in k[x_1,.,x_n]$ and $deg[P, x_m y_m] = degP$, $\forall m=1,.,n$.
2. $[Q, x_m y_m] \in k[y_1,.,y_n]$ and $deg[Q, x_m y_m] = degQ$, $\forall m=1,.,n$.

Proof: According to previous Remark 3.1, we have:

1. $[P, x_m y_m] = x_m [P, y_m] = -x_m P x_m'$. Then, $[P, x_m y_m] \in k[x_1,.,x_n]$ and $deg[P, x_m y_m] = degP$

2. Similarly, we obtain 2) □

Unlike the commutative case, the algebra of differential operators on Weyl algebra does not contain derivations, as the following result indicates:

iv. Theorem

For all $a \in A_n(k) \backslash k$, the multiplicative morphism l_a is not a differential operator on $A_n(k)$.

Proof: Let $a \in A_n(k)$.

- Case 1: for $a \in k$, we have $l_a = r_a \in D^0(A_n(k))$. Then, $L_k \subset D(A_n(k))$.
- Case 2: $a \in k[x_t]$ such that $deg(a) \geq 1$.

Since $[l_a, x_t y_t] = l_{[a, x_t y_t]}$. Then, from Lemma 3.1, we

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get
$[[[l_a, x_t y_t], x_t y_t], \ldots\ldots, x_t y_t] \notin R_{A_n(k)} = D^0(A_n(k))$.

Therefore, $l_a \notin D(A_n(k))$.

- Case 3: $a \in k[y_t]$ such that $\deg(a) \geq 1$. Similarly to case 2.
- Case 4: $a \in A\_\{n\}(k)\backslash k$.

According to Lemma 3.1, we can after t square brackets $[[[a, z_1], z_2], .. z_q \ldots\ldots z_t]$, where for all $q=1,\ldots,t$, $z_q \in \{x_1, .., x_n, y_1, \ldots, y_n\}$,
obtain an element $b \in k[x_t] \cup k[y_t]$. It follows that
$[[[l_a, z_1], z_2], .. z_q \ldots\ldots, z_t] = l_{[[[a,z_1],z_2],..zq\ldots\ldots,zt]} = l_b$.
From cases 2 and 3, we get $l_a \notin D(A_n(k))$. □.

v. Theorem

Non-zero derivation on $A_n(k)$ is not the differential operator.

Proof: Let $z \in A_n(k) = k\langle x_1, .., x_n, y_{1,\ldots,} y_n\rangle$ and d a non-zero derivation on $A_n(k)$.

According to Proposition 2.5, there exists $m \in A_n(k)$such that $d(z)=[m, z]$.

- Show that there exists $b_0 \in A_n(k)$ such that $d(b_0) \notin k$.

Suppose that
$d(z) \in k$, for all $z \in A_n(k)$ (2)
By taking $z = x_1$ and $z = y_1$, we obtain
$m = P + \alpha_1 y_1$, with $P \in k[x_1, .., x_n, y_2, .., y_n]$ and
$m = Q + \alpha_2 x_1$, with $Q \in k[x_2, .., x_n, y_1, .., y_n]$
Thus
$m = D + \alpha_2 x_1 + \alpha_1 y_1$ (3)
with $D \in k[x_2, .., x_n, y_2, .., y_n]$.
From (3), we get $d(x_1 y_1) = [m, x_1 y_1] = -\alpha_2 x_1 + \alpha_1 y_1$.
We deduce from (2) that $\alpha_2 = \alpha_1 = 0$.
Which means that $m \in A_{n-1}(k) = k\langle x_2, .., x_n, y_2, .., y_n\rangle$.
So on, we obtain $m \in k$ (Absurd) because d is non-zero.
Therefore, there exists $b_0 \in A_n(k)$ such that $d(b_0) \notin k$.
According to Theorem 3.1, $[d, b_0] = l_{d(b_0)} \notin D(A_n(k))$.
Hence, $d \notin D(A_n(k))$.

vi. Corollary

Let $m,n \in \mathbb{N}$.
$D^m(A_n(k))$ and $D(A_n(k))$ are not $A_n(k)$-modules on the left.
Proof: Indeed, according to previous Theorem 3.2, we have $au = l_a \circ u \notin D(A_n(k))$, for all $a \in A_n(k)\backslash k$ and $u \in D^m(A_n(k))$. □

B. Algebra of Differential Operators on a Noncommutative Algebra According yo M. Hazewinkel

Unlike the first approach, $D_n(A)$ and $D(A)$ have almost the same algebraic structures as the commutative case.

i. Proposition

Let $n \in \mathbb{N}$

1. $End_k(A)$ is a (A, A)-bimodule.
2. $End_k(A)$ is a (D(A),D(A))-bimodule.
3. $D_n(A)$ is a (A,A) –sub-bimodule of $End_k(A)$.
4. D(A) is a (A,A)-sub-bimodule of $End_k(A)$.

Proof

1. $End_k(A)$ is (A-A) bimodule with the external laws
   $\phi_1: A \times End_k(A) \longrightarrow End_k(A)$
   $(a,u) \mapsto l_a \circ u$
   and
   $\phi_2: End_k(A)) \times A \rightarrow End_k(A)$
   $(u,a) \mapsto u \circ l_a$
2. The following laws:
   $\phi_1: D(A) \times End_k(A) \longrightarrow End_k(A)$
   $(u,v) \mapsto u \circ v$
   and
   $\phi_2: End_k(A) \times D(A) \longrightarrow End_k(A)$
   $(v,u) \mapsto v \circ u$
   justify that D(A) is an (A,A)-subbimodule de $End_k(A)$.
3. Let $n \in \mathbb{N}$, $a \in A$ and $u \in DnA)$. We have $au = la \circ u \in Dn(A)$ and $ua = u \circ la \in Dn(A)$.
   Hence, $D_n(A)$ is an (A-A) sub-bimodule of $End_k(A)$.
4. From 3, we obtain 4. □

The following proposition specifies that the set D(A) of differential operators on a noncommutative algebra A, is indeed an algebra.

ii. Proposition

Let $m,n \in \mathbb{N}$

1. $id_A \in D_0(A)$.
2. $D_n(A) \subseteq D_{n+1}(A)$.
3. $D_m(A).D_n(A) \subseteq D_{m+n}(A)$.
4. D(A) is a subalgebra of $End_k(A)$.

Proof:

1. $id_A \in D_0(A)$ because $id_A = r_{1A}$
2. By induction on $n \in \mathbb{N}$. Let $u \in D_n(A)$.
   - For n=0, $D_0(A) = L_A R_A L_A$ .Therefore, there exists $a,b,c \in A$ such that $u = l_a \circ r_c \circ l_b$.
     Then, for all $s \in A$, we have
     $[u, s] = l_a \circ [g, s] \circ l_b + l_{as-sa} \circ g \circ l_b + l_a \circ g \circ l_{bs-sb} \in D_0(A)$.
     It follows that $u \in D_1'(A) \subseteq D_1(A)$. Hence, $D_0(A) \subseteq D_1(A)$.
   - Now, assume that the result is true for an integer $n \geq 0$ and prove it for n+1.
   - Let $u \in D_{n+1}(A)$.
     There exists a,b, $c \in A$ and $g \in D_{n+1}'(A)$ such that
     $u = l_a \circ g \circ l_b$.
     Then, for all $s \in A$, we get
     $[u, s] = l_a \circ [g, s] \circ l_a + l_{as-sa} \circ g \circ l_b + l_a \circ g \circ l_{bs-sb}$.
     Thus, $[u, s] \in D_{n+1}(A)$, for all $s \in A$. It follows that $u \in D_{n+2}'(A) \subseteq D_{n+2}(A)$.
   - We conclude that $D_n(A) \subseteq D_{n+1}(A)$, for all $n \in \mathbb{N}$.
3. See in [4] page 9-10.
4. From,1, 2 and 3, we get 4.

iii. Proposition

1. $D_0(A) = L_A R_A$
2. $Der_k(A) \subseteq D_1(A)$.

Proof: Let $\delta \in Der_k(A)$ and $a \in A$.
1) According to Definition 2.5.
2) Indeed $[\delta, a] = l_{\delta(a)} \in D_0(A)$. Then,
$\delta \in D_1'(A) \subseteq D_1(A)$. □

IV. CONCLUSION

In closing, we assert that the richness of the algebra of differential operators depends not only on its "commutative" nature but also on the methodological framework applied in its analysis. Consequently, Hazewinkel's approach proves to be the most effective for examining noncommutative algebras.

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