# An Elementary Proof for Fermat's Last Theorem using a Transformation Equation to Fermat's Equation

## P. N. Seetharaman



Abstract: Fermat's Last Theorem states that there are no positive integers x, y and z satisfying the equation  $x^n + y^n = z^n$ , where n is any integer > 2. Around 1637 Fermat proved that there are non-zero solutions to the above equation with n = 4. In the  $18^{th}$  century Euler treated the case n = 3, thereby reducing the proof for the case of a prime exponent  $\geq 5$  in this proof we hypothesize that r, s and t are positive integers satisfying the equation  $r^p + s^p = t^p$ , where p is any prime >3 and establish a contradiction. We use an Auxiliary equation  $x^3 + y^3 = z^3$  and create transformation equations. Solving the transformation equations we prove that only a trivial solution exists in the main equation  $r^p + s^p = t^p$ .

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#### I. INTRODUCTION

Around 1637, Pierre de Fermat, а French Mathematician, wrote in the margin of a book that the equation  $x^n + y^n = z^n$  had no solutions in positive integers, if n is an integer greater than 2. Although Fermat claimed to have found a general proof of his conjecture, he left no details of his proof. Leonhard Euler proved the theorem for the exponent n = 3, during 1770; Dirichlet, Legendre, Gabriel Lame proved the theorem for n = 5 and n= 7[1]. Around 1820, Sophie Germain proved the theorem partially, if  $\ell$  is a prime and  $(2\ell + 1)$  is also prime, in the Fermat's equation  $x^{\ell} + y^{\ell} = z^{\ell}$ , with exponent  $\ell$  does not divide (xyz). Ernst Kummer made the first substantial step in proving a part of Fermat's last theorem for many cases [2]. Hundreds of eminent mathematicians around the world in the last 350 years had contributed to Fermat's Last Theorem, by which number theory was developed rapidly. Finally in 1995 Professor Andrew Wiles, along with Professor Richard Taylor, proved the theorem completely and his paper was published in 'Annals of Mathematics'. His proof involved highly complicated and advanced number theory, which won him, Abel Award [3].

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Many mathematicians have analysed explained the theorem in all aspects [4]. In this proof, an alternative elementary proof for Fermat's Last theorem has been tried [5].

### **II. ASSUMPTIONS**

1) We presume that all r, s and t as positive integers satisfying the equation

$$r^p + s^p = t^p$$

where *p* is any prime > 3, and establish a contradiction. Obviously gcd(r, s, t) = 1;  $\sqrt{st}$  will be irrational, since both *s* & *t* cannot simultaneously be squares.

- 2) For supporting the proof in the equation  $r^p + s^p = t^p$ we use the Auxiliary equation  $x^3 + y^3 = z^3$ . Wherein, without loss of generality we can have both x and y as positive integer,  $z^3$  a positive integer; both z and  $z^2$ irrational. Since we are proving the theorem only in the main equations  $r^p + s^p = t^p$ , we have the choice of assigning the values as x = 17; y = 47;  $z^{3} = 17^{3} + 47^{3} =$  $8^2 \times 1699$ . The two equations  $x^3 + y^3 = z^3$  and  $r^p + s^p$  $= t^p$  are interconnected by means of using transformation equations having parameters called a, b, c, d, e and f.
- 3) We take *F*, *E* and *R* as odd primes with E = R; *F*, *E*, R will be coprimes to x, y,  $z^3$ , r, s, t and also r, s, t are coprime to x, y,  $z^3$ ; If not, we have the choice of assigning alternative values of x = 11; y = 53;  $z^3 = 11^3$  $+53^3 = 8^2 \times 2347$  so on.

Proof:

By trials we have created the following equations

$$\frac{a\sqrt{t} + b\sqrt{s}}{\sqrt{47}} \bigg)^2 + \left(\frac{c\sqrt{r^p} + d\sqrt{t}}{\sqrt{1699}}\right)^2 = \left(e\sqrt{E^{1/3}} + f\sqrt{F^{1/3}}\right)^2$$

and

$$\left(a\sqrt{t} - b\sqrt{E^{1/3}}\right)^2 + \left(c\sqrt{r} - d\sqrt{R^{1/3}}\right)^2 = \left(e\sqrt{s^p} - f\sqrt{17R^{5/3}}\right)^2$$
(1)

as the transformation equations of  $x^3 + y^3 = z^3$  and  $r^p + s^p$  $= t^p$  respectively, which holds good through the parameters called a, b, c, d, e and f. Here, x = 17; y = 47;  $z^3 = 17^3 + 47^3 =$  $8^2 \times 1699$  [6].

F, E and R are distinct odd primes, each coprime to each of x, y,  $z^3$ , r, s and t;

In this proof, we take r as coprime to 17 and 47, where x = 17 and y = 47. It not, we have the choice of assigning alternate values as x = 11, y = 53 and  $z^3 = 11^3 + 53^3 = 8^2 \times$ 2347, since we are proving the

theorem only in the main equation  $r^p + s^p = t^p$ .

From equation (1), we get

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$$a\sqrt{t} + b\sqrt{s} = \sqrt{47x^{3}} \quad \dots \quad (2)$$

$$a\sqrt{t^{p}} - b\sqrt{E^{1/3}} = \sqrt{r^{p}} \quad \dots \quad (3)$$

$$c\sqrt{r^{p}} + d\sqrt{t} = \sqrt{1699y^{3}} \quad \dots \quad (4)$$

$$c\sqrt{r} - d\sqrt{R^{1/3}} = \sqrt{s^{p}} \quad \dots \quad (5)$$

$$e\sqrt{E^{1/3}} + f\sqrt{F^{1/3}} = \sqrt{z^{3}} \quad \dots \quad (6)$$
and
$$e\sqrt{s^{p}} - f\sqrt{17R^{5/3}} = \sqrt{t^{p}} \quad \dots \quad (7)$$

Solving simultaneously (2) and (3), (4) and (5), (6) and (7), we get

$$\begin{aligned} a &= \left(\sqrt{47E^{1/3}x^3} + \sqrt{r^ps}\right) / \left(\sqrt{E^{1/3}t} + \sqrt{st^p}\right) \\ b &= \left(\sqrt{47x^3t^p} - \sqrt{r^pt}\right) / \left(\sqrt{E^{1/3}t} + \sqrt{st^p}\right) \\ c &= \left(\sqrt{1699R^{1/3}y^3} + \sqrt{s^pt}\right) / \left(\sqrt{R^{1/3}r^p} + \sqrt{rt}\right) \\ d &= \left(\sqrt{1699y^3r} - \sqrt{r^ps^p}\right) / \left(\sqrt{R^{1/3}r^p} + \sqrt{rt}\right) \\ e &= \left(\sqrt{17R^{5/3}z^3} + \sqrt{F^{1/3}t^p}\right) / \left(\sqrt{F^{1/3}s^p} + \sqrt{17E^{1/3}R^{5/3}}\right) \\ f &= \left(\sqrt{z^3s^p} - \sqrt{E^{1/3}t^p}\right) / \left(\sqrt{F^{1/3}s^p} + \sqrt{17E^{1/3}R^{5/3}}\right) \end{aligned}$$

From (3) & (7), we get

and

$$\sqrt{t^{p}} \times \sqrt{t^{p}} = \left(\sqrt{r^{p}} + b\sqrt{E^{1/3}}\right) \left(e\sqrt{s^{p}} - f\sqrt{17R^{5/3}}\right) / (a)$$
  
i.e.,  $t^{p} = \left\{ (e)\sqrt{r^{p}s^{p}} - (f)\sqrt{17R^{5/3}r^{p}} + (be)\sqrt{E^{1/3}s^{p}} - (bf)\sqrt{17E^{1/3}R^{5/3}} \right\} / (a)$ 

From (3) & (4), we get

$$\sqrt{r^{p}} \times \sqrt{r^{p}} = \left(a\sqrt{t^{p}} - b\sqrt{E^{1/3}}\right) \left(\sqrt{1699 y^{3}} - d\sqrt{t}\right) / (c)$$
  
i.e.,  $r^{p} = \left\{(a)\sqrt{1699 \times y^{3} t^{p}} - (ad)\sqrt{t^{p+1}} - (b)\sqrt{1699 E^{1/3} y^{3}} + (bd)\sqrt{E^{1/3} t}\right\} / (c)$ 

From (5) & (7), we get

$$\sqrt{s^{p}} \times \sqrt{s^{p}} = \left(c\sqrt{r} - d\sqrt{R^{1/3}}\right) \left(\sqrt{t^{p}} + f\sqrt{17R^{5/3}}\right) / (e)$$
  
i.e.,  $s^{p} = \left\{ (c)\sqrt{rt^{p}} + (cf)\sqrt{17R^{5/3}r} - (d)\sqrt{R^{1/3}t^{p}} - (df)R\sqrt{17} \right\} / (e)$ 

Substituting the above new values of  $t^p$ ,  $r^p$  and  $s^p$  in the equation  $t^p = r^p + s^p$  after multiplying both sides by  $\{ace\}$ , we get

$$\{ce\} \{(e)\sqrt{r^{p}s^{p}} - (f)\sqrt{17R^{5/3}r^{p}} + (be)\sqrt{E^{1/3}s^{p}} - (bf)\sqrt{17E^{1/3}R^{5/3}} \}$$

$$= (ae)\{(a)\sqrt{1699y^{3}t^{p}} - (ad)\sqrt{t^{p+1}} - (b)\sqrt{1699E^{1/3}y^{3}} + (bd)\sqrt{E^{1/3}t} \}$$

$$+ (ac)\{(c)\sqrt{rt^{p}} + (cf)\sqrt{17R^{5/3}r} - (d)\sqrt{R^{1/3}t^{p}} - (df)R\sqrt{17} \}$$

$$(8)$$

Our aim is to compute all rational terms in equation (8) and equate them on both sides by

$$\left\{ \left( \sqrt{E^{1/3}t} + \sqrt{st^{p}} \right)^{2} \left( \sqrt{R^{1/3}r^{p}} + \sqrt{rt} \right)^{2} \left( \sqrt{F^{1/3}s^{p}} + \sqrt{17E^{1/3}R^{5/3}} \right)^{2} \right\}$$

for freeing from denominators on the parameters *a*, *b*, *c*, *d*, *e* and *f* and again multiplying both sides by  $\sqrt{st}$  for getting some rational terms [7].

I term in LHS of equation (8), after multiplying by the respective terms and substituting for {
$$ce^2$$
}  
=  $\sqrt{r^p s^p} \left( E^{1/3}t + st^p + 2\sqrt{t^{p+1}}\sqrt{E^{1/3}s} \right) \left(\sqrt{R^{1/3}r^p} + \sqrt{rt} \right) \left(\sqrt{1699R^{1/3}y^3} + \sqrt{s^pt} \right)$   
 $\times \sqrt{st} \left( 17R^{5/3}z^3 + F^{1/3}t^p + 2\sqrt{17F^{1/3}R^{5/3}z^3t^p} \right)$ 

On multiplying by

$$\left\{\sqrt{r^{p}s^{p}}\left(E^{1/3}t\right)\sqrt{rt}\sqrt{st}\sqrt{s^{p}t}\left(17R^{5/3}z^{3}\right)\right\}$$

we get

$$\left(17E^{1/3}R^{5/3}z^3s^{p}t^2\right)\sqrt{st}\sqrt{r^{p+1}}\right\}$$

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which is irrational, since both s & t cannot simultaneously be squares.

II term in LHS of equation (8), after multiplying by the respective terms and substituting for  $\{c(ef)\}$ 

$$= \left(-\sqrt{17R^{5/3}r^{p}}\right)\left(E^{1/3}t + st^{p} + 2\sqrt{t^{p+1}\sqrt{E^{1/3}s}}\right)\left(\sqrt{R^{1/3}r^{p}} + \sqrt{rt}\right)\left(\sqrt{1699R^{1/3}y^{3}} + \sqrt{s^{p}t}\right)$$
  
  $\times \sqrt{st}\left(\sqrt{17R^{5/3}z^{3}} + \sqrt{F^{1/3}t^{p}}\right)\left(\sqrt{z^{3}s^{p}} - \sqrt{E^{1/3}t^{p}}\right)$   
*ring by*

On multiply ng by

$$\left\{ \left( -\sqrt{17R^{5/3}r^p} \right) \left( st^p \right) \sqrt{rt} \sqrt{st} \sqrt{1699R^{1/3}y^3} \sqrt{17R^{5/3}z^3} \left( -\sqrt{E^{1/3}t^p} \right) \right\}$$

we get

$$\left\{ \left( 17Rst^{p} \right) \sqrt{E^{1/3}R^{5/3}} \sqrt{1699z^{3}} \sqrt{r^{p+1}} \sqrt{t^{p+1}} \sqrt{y^{3}st} \right\}$$

which is irrational since *s* & *t* are coprime to y = 47 (See item (3) under Assumptions) [8]. III term in LHS of equation (8), after multiplying by the respective terms and substituting for  $\{bce^2\}$ 

$$=\sqrt{E^{1/3}s^{p}}\left(\sqrt{E^{1/3}t}+\sqrt{st^{p}}\right)\left(\sqrt{R^{1/3}r^{p}}+\sqrt{rt}\right)\left(\sqrt{1699R^{1/3}y^{3}}+\sqrt{s^{p}t}\right)$$
$$\times\sqrt{st}\left(\sqrt{47x^{3}t^{p}}-\sqrt{r^{p}t}\right)\left(17R^{5/3}z^{3}+F^{1/3}t^{p}+2\sqrt{17F^{1/3}R^{5/3}z^{3}t^{p}}\right)$$

On multiplying by

$$\left\{\sqrt{E^{1/3}s^p}\sqrt{E^{1/3}t}\sqrt{rt}\sqrt{s^pt}\sqrt{st}\left(-\sqrt{r^pt}\right)\left(17R^{5/3}z^3\right)\right\}$$

we get

$$\left\{ \left( -17E^{1/3}R^{5/3}z^3s^{p}t^2 \right) \sqrt{st}\sqrt{r^{p+1}} \right\}$$

which is irrational.

IV term in LHS of equation (8), after multiplying by the respective terms and substituting for  $\{bc(ef)\}$ 

$$= \left(-\sqrt{17E^{1/3}R^{5/3}}\right) \left(\sqrt{E^{1/3}t} + \sqrt{st^{p}}\right) \left(\sqrt{R^{1/3}r^{p}} + \sqrt{rt}\right) \left(\sqrt{1699R^{1/3}y^{3}} + \sqrt{s^{p}t}\right) \\ \times \sqrt{st} \left(\sqrt{47x^{3}t^{p}} - \sqrt{r^{p}t}\right) \left(\sqrt{17R^{5/3}z^{3}} + \sqrt{F^{1/3}t^{p}}\right) \left(\sqrt{z^{3}s^{p}} - \sqrt{E^{1/3}t^{p}}\right)$$

On multiplying by

$$\left\{ \left( -\sqrt{17}E^{1/3}R^{5/3} \right) \sqrt{st^{p}} \sqrt{R^{1/3}r^{p}} \sqrt{s^{p}t} \sqrt{st} \left( -\sqrt{r^{p}t} \right) \sqrt{17}R^{5/3}z^{3} \sqrt{z^{3}s^{p}} \right\}$$

we get

$$\left\{ \left( 17Rz^{3}r^{p}s^{p+1}t \right) \sqrt{E^{1/3}R^{5/3}} \sqrt{t^{p+1}} \right\}$$

which will be rational since E = R.

I term in RHS of equation (8), after multiplying by the respective terms and substituting for  $\{a^2e\}$ 

$$=\sqrt{1699 y^{3} t^{p}} \left(R^{1/3} r^{p} + rt + 2\sqrt{r^{p+1}} \sqrt{R^{1/3} t}\right) \left(\sqrt{F^{1/3} s^{p}} + \sqrt{17 E^{1/3} R^{5/3}}\right) \sqrt{st} \\ \times \left\{ \left(47 E^{1/3} x^{3}\right) + r^{p} s + 2\sqrt{47 E^{1/3} x^{3} r^{p} s} \right\} \left(\sqrt{17 R^{5/3} z^{3}} + \sqrt{F^{1/3} t^{p}}\right) \right\}$$
On multiplying by

On multiplying by

$$\sqrt{1699y^3t^p} \left(2\sqrt{r^{p+1}}\sqrt{R^{1/3}t}\right)\sqrt{17E^{1/3}R^{5/3}}\sqrt{st}(r^p s)\sqrt{17R^{5/3}z^3}$$

we get

$$\left\{ (2 \times 17 R r^p s) \sqrt{E^{1/3} R^{5/3}} \sqrt{(rt)^{p+1} \sqrt{1699 z^3}} \sqrt{y^3 s t} \right\}$$

which will be irrational, since both s and t are coprime to y = 47 (See Assumptions) [9]. II term in RHS of equation (8), after multiplying by the respective terms and substituting for  $\{(a^2de)\}$ 

$$= \left(-\sqrt{t^{p+1}}\right) \left(\sqrt{R^{1/3}r^{p}} + \sqrt{rt}\right) \left(\sqrt{F^{1/3}s^{p}} + \sqrt{17E^{1/3}R^{5/3}}\right) \left(\sqrt{1669y^{3}r} - \sqrt{r^{p}s^{p}}\right) \sqrt{st} \\ \times \left\{ \left(47E^{1/3}x^{3}\right) + r^{p}s + 2\sqrt{47E^{1/3}x^{3}r^{p}s} \right\} \left(\sqrt{17R^{5/3}z^{3}} + \sqrt{F^{1/3}t^{p}}\right) \left(\sqrt{1669y^{3}r^{2}} + \sqrt{1669y^{3}r^{2}}\right) \left(\sqrt{1$$

On multiplying by

$$\left\{ \left( -\sqrt{t^{p+1}} \right) \sqrt{R^{1/3} r^p} \sqrt{17 E^{1/3} R^{5/3}} \sqrt{1669 y^3 r} \sqrt{st} \left( r^p s \right) \sqrt{17 R^{5/3} z^3} \right\}$$

$$\left\{ \left( -17 R r^p s \right) \sqrt{E^{1/3} R^{5/3}} \sqrt{1669 z^3} \sqrt{\left( rt \right)^{p+1}} \sqrt{y^3 st} \right\}$$

we get

$$\left\{ \left(-17Rr^{p}s\right)\sqrt{E^{1/3}R^{5/3}}\sqrt{1669z^{3}}\sqrt{\left(rt\right)^{p+1}}\sqrt{y^{3}st}\right\}$$

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which is irrational since *s* and *t* are coprime to y = 47 (See Assumptions).

III term in RHS of equation (8), after multiplying by the respective terms and substituting for  $\{(ab)e\}$ 

$$= \left(-\sqrt{1699 y^{3} t^{p}}\right) \left(R^{1/3} r^{p} + rt + 2\sqrt{r^{p+1}} \sqrt{R^{1/3} t}\right) \left(\sqrt{F^{1/3} s^{p}} + \sqrt{17 E^{1/3} R^{5/3}}\right) \sqrt{st} \\ \times \left(\sqrt{47 E^{1/3} x^{3}} + \sqrt{r^{p} s}\right) \left(\sqrt{47 x^{3} t^{p}} - \sqrt{r^{p} t}\right) \left(\sqrt{17 R^{5/3} z^{3}} + \sqrt{F^{1/3} t^{p}}\right) \left(\sqrt{17 R^{5/3} z^{3}} + \sqrt{F^{1/3}$$

On multiplying by

$$\left(\left(-\sqrt{1699\,y^{3}E^{1/3}}\right)(rt)\sqrt{17E^{1/3}R^{5/3}}\sqrt{st}\sqrt{r^{p}s}\left(-\sqrt{r^{p}t}\right)\sqrt{17R^{5/3}z^{3}}\right)$$

we get

$$\left\{ \left( 17E^{1/3}R^{5/3}r^{p+1}st^2 \right) \sqrt{1669z^3} \sqrt{y^3} \right\}$$

which is irrational, since y = 47.

IV term in RHS of equation (8), after multiplying by the respective terms and substituting for  $\{(ab)de\}$  $=\sqrt{E^{1/3}t}\left(\sqrt{R^{1/3}r^{p}}+\sqrt{rt}\right)\left(\sqrt{F^{1/3}s^{p}}+\sqrt{17E^{1/3}R^{5/3}}\right)\left(\sqrt{1669y^{3}r}-\sqrt{r^{p}s^{p}}\right)\sqrt{st}$ 

$$\times \left(\sqrt{47E^{1/3}x^{3}} + \sqrt{r^{p}s}\right) \left(\sqrt{47x^{3}t^{p}} - \sqrt{r^{p}t}\right) \left(\sqrt{17R^{5/3}z^{3}} + \sqrt{F^{1/3}t^{p}}\right)$$

On multiplying by

$$= \sqrt{E^{1/3}t}\sqrt{rt}\sqrt{17E^{1/3}R^{5/3}}\sqrt{st}\sqrt{1669y^3r}\sqrt{r^ps}\left(-\sqrt{r^pt}\right)\sqrt{17R^{5/3}z^3}$$

we get

$$\left\{-\left(17E^{1/3}R^{5/3}r^{p+1}st^2\right)\sqrt{1669z^3}\sqrt{y^3}\right\}$$

which is irrational. V term in RHS of equation (8), after multiplying by the respective terms and substituting for  $\{ac^2\}$ 

$$= \sqrt{rt^{p}} \left( \sqrt{E^{1/3}t} + \sqrt{st^{p}} \right) \left( F^{1/3}s^{p} + 17E^{1/3}R^{5/3} + 2\sqrt{17F^{1/3}s^{p}E^{1/3}R^{5/3}} \right) \sqrt{st} \\ \times \left( \sqrt{47E^{1/3}x^{3}} + \sqrt{r^{p}s} \right) \left\{ \left( 1699R^{1/3}y^{3} \right) + \left( s^{p}t \right) + \left( 2\sqrt{1699R^{1/3}y^{3}s^{p}t} \right) \right\}$$

On multiplying by

$$\left\{\sqrt{rt^{p}}\sqrt{st^{p}}\left(17E^{1/3}R^{5/3}\right)\sqrt{st}\sqrt{r^{p}s}\left(s^{p}t\right)\right\}$$

we get

$$\left\{ \left( 17E^{1/3}R^{5/3} \right) \left( st \right)^{p+1} \sqrt{r^{p+1}} \sqrt{st} \right\}$$

which is irrational, (since  $\sqrt{st}$  is irrational).

VI term in RHS of equation (8), after multiplying by the respective terms and substituting for { 
$$ac^2f$$
 }  
= $(\sqrt{17R^{5/3}r})(\sqrt{E^{1/3}t} + \sqrt{st^p})(\sqrt{F^{1/3}s^p} + \sqrt{17E^{1/3}R^{5/3}})\sqrt{st}(\sqrt{z^3s^p} - \sqrt{E^{1/3}t^p})$   
 $\times (\sqrt{47E^{1/3}x^3} + \sqrt{r^ps})\{(1699R^{1/3}y^3) + (s^pt) + (2\sqrt{1699R^{1/3}y^3s^pt})\}$ 

$$\left\{\sqrt{17R^{5/3}r}\sqrt{st^{p}}\sqrt{17E^{1/3}R^{5/3}}\sqrt{st}\sqrt{r^{p}s}\left(s^{p}t\right)\left(-\sqrt{E^{1/3}t^{p}}\right)\right\}$$

we get

$$\left\{-\left(17E^{1/3}R^{5/3}\right)\left(s^{p+1}t^{p+1}\right)\sqrt{r^{p+1}}\sqrt{st}\right\}$$

which is irrational.

VII term in LHS of equation (8), after multiplying by the respective terms and substituting for 
$$\{a(cd)\}$$
  
= $\left(-\sqrt{R^{1/3}t^{p}}\right)\left(\sqrt{E^{1/3}t} + \sqrt{st^{p}}\right)\left(F^{1/3}s^{p} + 17E^{1/3}R^{5/3} + 2\sqrt{17F^{1/3}s^{p}E^{1/3}R^{5/3}}\right)\sqrt{st}$   
 $\times\left(\sqrt{47E^{1/3}x^{3}} + \sqrt{r^{p}s}\right)\left(\sqrt{1699R^{1/3}y^{3}} + \sqrt{s^{p}t}\right)\left(\sqrt{1699y^{3}r} - \sqrt{r^{p}s^{p}}\right)$ 

There is no rational part in this term.

VIII term in LHS of equation (8), after multiplying by the respective terms and substituting for  $\{a(cd)f\}$  $= \left(-R\sqrt{17}\right) \left(\sqrt{E^{1/3}t} + \sqrt{st^{p}}\right) \left(\sqrt{F^{1/3}s^{p}} + \sqrt{17E^{1/3}R^{5/3}}\right) \sqrt{st} \left(\sqrt{7^{3}s^{p}} - \sqrt{E^{1/3}t^{p}}\right)$ 

$$\times \left(\sqrt{47E^{1/3}x^3} + \sqrt{r^ps}\right) \left(\sqrt{1699R^{1/3}y^3} + \sqrt{s^pt}\right) \left(\sqrt{1699y^3r} - \sqrt{r^ps^p}\right)$$
  
On multiplying by  
$$\left\{ \left(-R\sqrt{17}\right)\sqrt{st^p}\sqrt{17E^{1/3}R^{5/3}}\sqrt{st}\sqrt{z^3s^p}\sqrt{r^ps}\sqrt{s^pt}\sqrt{1699y^3r} \right\}$$

С

$$\left\{ \left(-R\sqrt{17}\right)\sqrt{st^{p}}\sqrt{17E^{1/3}R^{5/3}}\sqrt{st}\sqrt{z^{3}s^{p}}\sqrt{r^{p}s}\sqrt{s^{p}t}\sqrt{1699y^{3}r}\right\}$$

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we get

$$\left\{-(17R)\sqrt{E^{1/3}R^{5/3}}\left(s^{p+1}\right)\sqrt{r^{p+1}}\sqrt{t^{p+1}}\sqrt{1699z^3}\sqrt{y^3st}\right\}$$

which is irrational.

Sum of all rational terms in LHS of equation (8)

 $= \left\{ \left( 17Rz^{3}r^{p}s^{p+1}t \right) \sqrt{E^{1/3}R^{5/3}} \sqrt{t^{p+1}} \right\}$  (vide IV term)

There is no rational terms on RHS of equation (8).

Equating the rational terms on both sides of equation (8), we get

$$(17Rz^3)\sqrt{E^{1/3}R^{5/3}}\left(r^ps^{p+1}t\sqrt{t^{p+1}}\right) = 0$$

Dividing both sides by

$$(17Rz^3)\sqrt{E^{1/3}R^{5/3}}$$

we get

 $\left(r^{p}s^{p+1}t\sqrt{t^{p+1}}\right) = 0$ That is, either r = 0; or s = 0; or t = 0.

This contradicts our hypothesis that all *r*, *s* and *t* are nonzero integers in the equation  $r^p + s^p = t^p$ , with *p* any prime >3, thus proving that only a trivial solution exists in the equation  $r^p + s^p = t^p$ ,

#### **III. CONCLUSION**

In this proof equation (8) has obtained from the two transformation equations for  $x^3 + y^3 = z^3$  and  $r^p + s^p = t^p$ . by using the equivalent values of  $r^p$ ,  $s^p \& t^p$ , which are substituted in the equation  $r^p + s^p = t^p$ . Hence the result *rst* = 0, that we get from equation (8) proves that there is no non-zero integer solutions exists in the equation  $r^p + s^p = t^p$ .

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