

An Elementary Proof for Fermat's Last Theorem using Three Distinct Odd Primes F, E and R

P. N. Seetharaman



Abstract: In number theory, Fermat's Last Theorem states that no three positive integers a, b and c satisfy the equation $a^n + b^n = c^n$ where n is any integer > 2 . Fermat and Euler had already proved that there are no integral solutions to the equations $x^3 + y^3 = z^3$ and $x^4 + y^4 = z^4$. Hence, it would suffice to prove the theorem for the index $n = p$, where p is any prime greater than 3. In this proof, we have hypothesised that r, s , and t are positive integers in the equation $rp + sp = tp$, where p is any prime greater than 3, and we prove the theorem using the method of contradiction. We have used auxiliary equations $x^3 + y^3 = z^3$, along with the primary equation $rp + sp = tp$, which are connected using a transformation equation through the parameters. Solving the transformation equations, we get the result $rst = 0$, showing that only a trivial solution exists in the central equation.

Keywords: Transformation Equations, Two Fermat's Equations. 2010 Mathematics Subject Classification 2010: 11A-XX.

Abbreviations:

LHS: Left Hand Side
RHS: Right Hand Side

I. INTRODUCTION

During 1637, the French mathematician Pierre de Fermat conjectured in the margin of a book that the equation $x^n + y^n = z^n$ has no integral solutions for x, y and z , where $n > 2$ [1]. He mentioned therein that he had found a marvellous solution to $x^n + y^n = z^n$, with $n > 2$, but the margin was too narrow to contain it. However, his proof is available only for $x^4 + y^4 = z^4$ using infinite decent method [2]. Subsequently, Euler and others proved the theorem for $x^3 + y^3 = z^3$. Later on, Sophie Germain proved the theorem for a general case, and subsequently E.E. [3]. Kummer proved the theorem for regular primes [4]. Many mathematicians contributed to the development of this theorem, resulting in significant advancements in number theory. Mathematicians have found a close relationship between Fermat's Last Theorem and elliptic curves. Finally, in 1995, Andrew Wiles proved the theorem completely. Many mathematicians have analysed and explained the theorem in all aspects. In this paper, we attempt an alternative elementary proof of Fermat's Last Theorem.

II. ASSUMPTIONS

- A. We initially hypothesize that all r, s and t are non-zero integers satisfying the equation $r^p + s^p = t^p$ where p is any prime > 3 , and establish a contradiction in this proof. We can have $\gcd(r, s, t) = 1$
- B. We employ the Auxiliary equation $x^3 + y^3 = z^3$, along with the primary equation $rp + sp = tp$. Since we are proving the theorem only in the central equation, we have the choice of assigning suitable numerical values for x, y and z^3 . Without loss of generality, we can assume that x and y are positive integers, z^3 is a positive integer, and both z and z^2 are irrational. In this proof, we have assigned the values as $x = 53, y = 11, z^3 = 53^3 + 11^3 = 8^2 \times 2347$. We have created transformation equations for the above two equations and linked them through parameters a, b, c, d, e , and f .
- C. F, E and R are distinct odd primes; each is coprime to x, y, z^3 , and r, s and t .

Proof. By random trials, we have created the following equations,

$$(a\sqrt{t^p} + b\sqrt{11E^{1/3}})^2 + (c\sqrt{2347} + d\sqrt{F})^2 = (e\sqrt{E^{5/3}} + f\sqrt{53})^2 \quad (1)(i)$$

and

$$(a\sqrt{F} - b\sqrt{E})^2 + (c\sqrt{r^p} - d\sqrt{R^{5/3}})^2 = (e\sqrt{s^p} - f\sqrt{R^{1/3}})^2 \quad (1)(ii)$$

as the transformation equations of $x^3 + y^3 = z^3$ and $r^p + s^p = t^p$ respectively, through the parameters called a, b, c, d, e and f . Here $x = 53; y = 11$; and $z^3 = x^3 + y^3 = 53^3 + 11^3 = 8^2 \times 2347$. F, E and R are distinct odd primes, each coprime to x, y, z^3, r, s and t .

From equations (1-i) and (1-ii), we get

$$a\sqrt{t^p} + b\sqrt{11E^{1/3}} = \sqrt{x^3} \quad (2)$$

$$a\sqrt{F} - b\sqrt{E} = \sqrt{r^p} \quad (3)$$

$$c\sqrt{2347} + d\sqrt{F} = \sqrt{y^3} \quad (4)$$

$$c\sqrt{r^p} - d\sqrt{R^{5/3}} = \sqrt{s^p} \quad (5)$$

$$e\sqrt{E^{5/3}} + f\sqrt{53} = \sqrt{z^3} \quad (6)$$

$$\text{and } e\sqrt{s^p} - f\sqrt{R^{1/3}} = \sqrt{t^p} \quad (7)$$

Solving simultaneously (2) and (3), (4) and (5), (6) and (7), we get

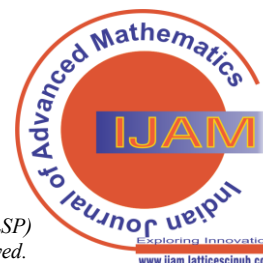
$$a = \left(\sqrt{Ex^3} + \sqrt{11E^{1/3}r^p} \right) / \left(\sqrt{Et^p} + \sqrt{11FE^{1/3}} \right)$$

Manuscript received on 09 February 2025 | First Revised Manuscript received on 13 February 2025 | Second Revised Manuscript received on 20 March 2025 | Manuscript Accepted on 15 April 2025 | Manuscript published on 30 April 2025.

*Correspondence Author(s)

P.N. Seetharaman*, Retired Executive Engineer, Energy Conservation Cell, Tamil Nadu State Electricity Board, Anna Salai, Chennai (Tamil Nadu), India. Email ID: palamadaiseetharaman@gmail.com, ORCID ID: [0000-0002-4615-1280](https://orcid.org/0000-0002-4615-1280)

© The Authors. Published by Lattice Science Publication (LSP). This is an open access article under the CC-BY-NC-ND license <http://creativecommons.org/licenses/by-nc-nd/4.0/>



$$b = \left(\sqrt{Fx^3} - \sqrt{r^p t^p} \right) / \left(\sqrt{Et^p} + \sqrt{11FE^{1/3}} \right)$$

$$c = \left(\sqrt{R^{5/3} y^3} + \sqrt{Fs^p} \right) / \left(\sqrt{Fr^p} + \sqrt{2347R^{5/3}} \right)$$

$$d = \left(\sqrt{r^p y^3} - \sqrt{2347s^p} \right) / \left(\sqrt{Fr^p} + \sqrt{2347R^{5/3}} \right)$$

$$e = \left(\sqrt{R^{1/3} z^3} + \sqrt{53t^p} \right) / \left(\sqrt{53s^p} + \sqrt{E^{5/3} R^{1/3}} \right)$$

and $f = \left(\sqrt{z^3 s^p} - \sqrt{E^{5/3} t^p} \right) / \left(\sqrt{53s^p} + \sqrt{E^{5/3} R^{1/3}} \right)$

From (3) & (5), we have

$$\sqrt{r^p} \times \sqrt{r^p} = \left(a\sqrt{F} - b\sqrt{E} \right) \left(\sqrt{s^p} + d\sqrt{R^{5/3}} \right) / (c)$$

i.e., $r^p = \left\{ (a)\sqrt{Fs^p} + (ad)\sqrt{FR^{5/3}} - (b)\sqrt{Es^p} - (bd)\sqrt{ER^{5/3}} \right\} / (c)$

From (5) & (7), we get

$$\sqrt{s^p} \times \sqrt{s^p} = \left(c\sqrt{r^p} - d\sqrt{R^{5/3}} \right) \left(\sqrt{t^p} + f\sqrt{R^{1/3}} \right) / (e)$$

i.e., $s^p = \left\{ (c)\sqrt{r^p t^p} + (cf)\sqrt{R^{1/3} r^p} - (d)\sqrt{R^{5/3} t^p} - (df)R \right\} / (e)$

From (2) & (7), we get

$$\sqrt{t^p} \times \sqrt{t^p} = \left(\sqrt{x^3} - b\sqrt{11E^{1/3}} \right) \left(e\sqrt{s^p} - f\sqrt{R^{1/3}} \right) / (a)$$

i.e., $t^p = \left\{ (e)\sqrt{x^3 s^p} - (f)\sqrt{R^{1/3} x^3} - (be)\sqrt{11E^{1/3} s^p} + (bf)\sqrt{11E^{1/3} R^{1/3}} \right\} / (a)$

On substituting the above equivalent values of r^p, s^p and t^p in Fermat's equation, $t^p = r^p + s^p$ after multiplying both sides by $\{(ace)\}$, we get

$$\{(ce)\} \left\{ (e)\sqrt{x^3 s^p} - (f)\sqrt{R^{1/3} x^3} - (be)\sqrt{11E^{1/3} s^p} + (bf)\sqrt{11E^{1/3} R^{1/3}} \right\}$$

$$= \{ae\} \left\{ (a)\sqrt{Fs^p} + (ad)\sqrt{FR^{5/3}} - (b)\sqrt{Es^p} - (bd)\sqrt{ER^{5/3}} \right\}$$

$$+ \{(ac)\} \left\{ (c)\sqrt{r^p t^p} + (cf)\sqrt{R^{1/3} r^p} - (d)\sqrt{R^{5/3} t^p} - (df)R \right\} \quad (8)$$

Our purpose is to compute all rational terms in equation (8) and equate them on both sides, after multiplying both sides by

$$\left(\sqrt{Et^p} + \sqrt{11FE^{1/3}} \right)^2 \left(\sqrt{Fr^p} + \sqrt{2347R^{5/3}} \right)^2 \left(\sqrt{53s^p} + \sqrt{E^{5/3} R^{1/3}} \right)^2$$

for freeing from denominators on the parameters a, b, c, d, e and f , and again multiplying both sides by $(E^{1/3} \sqrt{y})$ For obtaining some rational terms, as worked out hereunder, term by term.

I term in LHS of equation (8), after multiplying by the respective terms and substituting for $\{ce^2\}$

$$= \sqrt{x^3 s^p} \left\{ (Et^p) + (11FE^{1/3}) + 2E^{2/3} \sqrt{11Ft^p} \right\} \left(\sqrt{Fr^p} + \sqrt{2347R^{5/3}} \right)$$

$$\times (E^{1/3} \sqrt{y}) \left(\sqrt{R^{5/3} y^3} + \sqrt{Fs^p} \right) \left\{ (R^{1/3} z^3) + (53t^p) + 2\sqrt{R^{1/3} z^3} \sqrt{53t^p} \right\}$$

On multiplying by

$$\left\{ \sqrt{x^3 s^p} \left(2E^{2/3} \sqrt{11Ft^p} \right) \sqrt{2347R^{5/3}} \left(E^{1/3} \sqrt{y} \right) \sqrt{Fs^p} \left(2\sqrt{R^{1/3} z^3} \sqrt{53t^p} \right) \right\}$$

We get

$$\left\{ (4FERS^p t^p) \sqrt{53x^3} \sqrt{11y} \sqrt{2347z^3} \right\}$$

II term in LHS of equation (8), after multiplying by the respective terms and substituting for $\{c(ef)\}$

$$= \left(-\sqrt{R^{1/3} x^3} \right) \left\{ (Et^p) + (11FE^{1/3}) + 2E^{2/3} \sqrt{11Ft^p} \right\} \left(\sqrt{Fr^p} + \sqrt{2347R^{5/3}} \right)$$

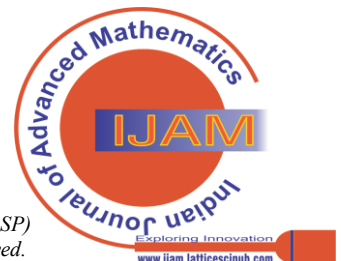
$$\times (E^{1/3} \sqrt{y}) \left(\sqrt{R^{5/3} y^3} + \sqrt{Fs^p} \right) \left(\sqrt{R^{1/3} z^3} + \sqrt{53t^p} \right) \left(\sqrt{z^3 s^p} - \sqrt{E^{5/3} t^p} \right)$$

Rational part in this term

$$= \left\{ \left(-\sqrt{R^{1/3} x^3} \right) \left(2E^{2/3} \sqrt{11Ft^p} \right) \sqrt{2347R^{5/3}} \left(E^{1/3} \sqrt{y} \right) \sqrt{Fs^p} \sqrt{53t^p} \sqrt{z^3 s^p} \right\}$$

$$= \left\{ - (2FERS^p t^p) \sqrt{53x^3} \sqrt{11y} \sqrt{2347z^3} \right\}$$

III term in LHS of equation (8), after multiplying by the respective terms and substituting



for $\{bce^2\}$

$$= \left(-\sqrt{11E^{1/3}s^p}\right)\left(\sqrt{Et^p} + \sqrt{11FE^{1/3}}\right)\left(\sqrt{Fr^p} + \sqrt{2347R^{5/3}}\right)\left(\sqrt{Fx^3} - \sqrt{r^p t^p}\right) \\ \times \left(E^{1/3}\sqrt{y}\right)\left(\sqrt{R^{5/3}y^3} + \sqrt{Fs^p}\right)\left\{\left(R^{1/3}z^3\right) + \left(53t^p\right) + 2\sqrt{R^{1/3}z^3}\sqrt{53t^p}\right\}$$

Rational part in this term

$$= \left\{\left(-\sqrt{11E^{1/3}s^p}\right)\sqrt{Et^p}\sqrt{2347R^{5/3}}\sqrt{Fx^3}\left(E^{1/3}\sqrt{y}\right)\sqrt{Fs^p}\left(2\sqrt{R^{1/3}z^3}\sqrt{53t^p}\right)\right\} \\ = \left\{-\left(2FERs^p t^p\right)\sqrt{53x^3}\sqrt{11y}\sqrt{2347z^3}\right\}$$

IV term in LHS of equation (8), after multiplying by the respective terms and substituting for $\{bc(ef)\}$

$$= \left(\sqrt{11E^{1/3}R^{1/3}}\right)\left(\sqrt{Et^p} + \sqrt{11FE^{1/3}}\right)\left(\sqrt{Fr^p} + \sqrt{2347R^{5/3}}\right)\left(\sqrt{Fx^3} - \sqrt{r^p t^p}\right) \\ \times \left(E^{1/3}\sqrt{y}\right)\left(\sqrt{R^{5/3}y^3} + \sqrt{Fs^p}\right)\left(\sqrt{R^{1/3}z^3} + \sqrt{53t^p}\right)\left(\sqrt{z^3 s^p} - \sqrt{E^{5/3}t^p}\right)$$

Rational part in this term

$$= \left\{\sqrt{11E^{1/3}R^{1/3}}\sqrt{Et^p}\sqrt{2347R^{5/3}}\sqrt{Fx^3}\left(E^{1/3}\sqrt{y}\right)\sqrt{Fs^p}\sqrt{53t^p}\sqrt{z^3 s^p}\right\} \\ = \left\{\left(FERs^p t^p\right)\sqrt{53x^3}\sqrt{11y}\sqrt{2347z^3}\right\}$$

I term in RHS of equation (8), after multiplying by the respective terms and substituting for $\{a^2e\}$

$$= \sqrt{Fs^p}\left\{\left(2347R^{5/3}\right) + \left(Fr^p\right) + 2\sqrt{2347R^{5/3}}\sqrt{Fr^p}\right\}\left(\sqrt{E^{5/3}R^{1/3}} + \sqrt{53s^p}\right) \\ \times \left(E^{1/3}\sqrt{y}\right)\left(\sqrt{R^{1/3}z^3} + \sqrt{53t^p}\right)\left\{\left(Ex^3\right) + \left(11E^{1/3}r^p\right) + 2E^{2/3}\sqrt{11x^3 r^p}\right\}$$

Rational part in this term

$$= \left\{\sqrt{Fs^p}\left(2\sqrt{2347R^{5/3}}\sqrt{Fr^p}\right)\sqrt{53s^p}\left(E^{1/3}\sqrt{y}\right)\sqrt{R^{1/3}z^3}\left(2E^{2/3}\sqrt{11x^3 r^p}\right)\right\} \\ = \left\{\left(4FERr^p s^p\right)\sqrt{53x^3}\sqrt{11y}\sqrt{2347z^3}\right\}$$

II term in RHS of equation (8), after multiplying by the respective terms and substituting for $\{(a^2de)\}$

$$= \sqrt{FR^{5/3}}\left(\sqrt{2347R^{5/3}} + \sqrt{Fr^p}\right)\left(\sqrt{E^{5/3}R^{1/3}} + \sqrt{53s^p}\right)\left(E^{1/3}\sqrt{y}\right) \\ \times \left\{\left(Ex^3\right) + \left(11E^{1/3}r^p\right) + 2E^{2/3}\sqrt{11x^3 r^p}\right\}\left(\sqrt{r^p y^3} - \sqrt{2347s^p}\right)\left(\sqrt{R^{1/3}z^3} + \sqrt{53t^p}\right)$$

Rational part in this term

$$= \left\{\sqrt{FR^{5/3}}\sqrt{Fr^p}\sqrt{53s^p}\left(E^{1/3}\sqrt{y}\right)\left(2E^{2/3}\sqrt{11x^3 r^p}\right)\left(-\sqrt{2347s^p}\right)\sqrt{R^{1/3}z^3}\right\}$$

We get

$$\left\{-\left(2FERr^p s^p\right)\sqrt{53x^3}\sqrt{11y}\sqrt{2347z^3}\right\}$$

III term in RHS of equation (8), after multiplying by the respective terms and substituting for $\{(ab)e\}$

$$= \left(-\sqrt{Es^p}\right)\left\{\left(2347R^{5/3}\right) + \left(Fr^p\right) + 2\sqrt{2347R^{5/3}}\sqrt{Fr^p}\right\}\left(\sqrt{E^{5/3}R^{1/3}} + \sqrt{53s^p}\right) \\ \times \left(E^{1/3}\sqrt{y}\right)\left(\sqrt{R^{1/3}z^3} + \sqrt{53t^p}\right)\left(\sqrt{Ex^3} + \sqrt{11E^{1/3}r^p}\right)\left(\sqrt{Fx^3} - \sqrt{r^p t^p}\right)$$

Rational part in this term

$$= \left\{\left(-\sqrt{Es^p}\right)\left(2\sqrt{2347R^{5/3}}\sqrt{Fr^p}\right)\sqrt{53s^p}\left(E^{1/3}\sqrt{y}\right)\sqrt{R^{1/3}z^3}\sqrt{11E^{1/3}r^p}\sqrt{Fx^3}\right\} \\ = \left\{-\left(2FERr^p s^p\right)\sqrt{53x^3}\sqrt{11y}\sqrt{2347z^3}\right\}$$

IV term in RHS of equation (8), after multiplying by the respective terms and substituting for $\{(ab)de\}$

$$= \left(-\sqrt{ER^{5/3}}\right)\left(\sqrt{2347R^{5/3}} + \sqrt{Fr^p}\right)\left(\sqrt{E^{5/3}R^{1/3}} + \sqrt{53s^p}\right)\left(E^{1/3}\sqrt{y}\right) \\ \times \left(\sqrt{Ex^3} + \sqrt{11E^{1/3}r^p}\right)\left(\sqrt{Fx^3} - \sqrt{r^p t^p}\right)\left(\sqrt{r^p y^3} - \sqrt{2347s^p}\right)\left(\sqrt{R^{1/3}z^3} + \sqrt{53t^p}\right)$$

Rational part in this term

$$= \left\{\left(-\sqrt{ER^{5/3}}\right)\sqrt{Fr^p}\sqrt{53s^p}\left(E^{1/3}\sqrt{y}\right)\sqrt{11E^{1/3}r^p}\sqrt{Fx^3}\left(-\sqrt{2347s^p}\right)\sqrt{R^{1/3}z^3}\right\}$$



$$= \left\{ (FERr^p s^p) \sqrt{53x^3} \sqrt{11y} \sqrt{2347z^3} \right\}$$

V term in RHS of equation (8), after multiplying by the respective terms and substituting for $\{ac^2\}$

$$= \sqrt{r^p t^p} \left(\sqrt{Et^p} + \sqrt{11FE^{1/3}} \right) \left\{ (E^{5/3} R^{1/3}) + (53s^p) + 2\sqrt{E^{5/3} R^{1/3}} \sqrt{53s^p} \right\} \\ \times \left(E^{1/3} \sqrt{y} \right) \left(\sqrt{Ex^3} + \sqrt{11E^{1/3} r^p} \right) \left\{ (R^{5/3} y^3) + (Fs^p) + 2\sqrt{R^{5/3} y^3} \sqrt{Fs^p} \right\}$$

(i) Rational part in this term

$$= \left\{ \sqrt{r^p t^p} \sqrt{Et^p} (53s^p) (E^{1/3} \sqrt{y}) \sqrt{11E^{1/3} r^p} (Fs^p) \right\} \\ = \left\{ (53 \times FERr^p s^{2p} t^p) \sqrt{11y} \right\}$$

(ii) Further, multiplying by

$$= \left\{ \sqrt{r^p t^p} \sqrt{Et^p} (E^{5/3} R^{1/3}) (E^{1/3} \sqrt{y}) \sqrt{Ex^3} (R^{5/3} y^3) \right\} \\ = \left\{ (E^3 R^2 xy^3 t^p) \sqrt{r^p xy} \right\}$$

Which will be irrational if r is coprime to $x = 53$ and $y = 11$; if not, we have the choice of assigning alternative values for x and y such that $x = 17$; $y = 47$, $z^3 = 17^3 + 47 = 8^2 \times 1699$, such that r is coprime to 17 and 47.

The VI term in the RHS of equation (8), after multiplying by the respective terms and substituting for $\{ac^2f\}$, is

$$= \sqrt{R^{1/3} r^p} \left(\sqrt{Et^p} + \sqrt{11EF^{1/3}} \right) \left(\sqrt{E^{5/3} R^{1/3}} + \sqrt{53s^p} \right) (E^{1/3} \sqrt{y}) \\ \times \left(\sqrt{Ex^3} + \sqrt{11E^{1/3} r^p} \right) \left\{ (R^{5/3} y^3) + (Fs^p) + 2\sqrt{R^{5/3} y^3} \sqrt{Fs^p} \right\} \left(\sqrt{z^3 s^p} - \sqrt{E^{5/3} t^p} \right)$$

On multiplying by

$$\left\{ \sqrt{R^{1/3} r^p} \sqrt{Et^p} \sqrt{E^{5/3} R^{1/3}} (E^{1/3} \sqrt{y}) \sqrt{11E^{1/3} r^p} (R^{5/3} y^3) (-\sqrt{E^{5/3} t^p}) \right\}$$

We get

$$\left\{ -(E^{8/3} R^2 y^3 r^p t^p) \sqrt{11y} \right\}$$

Which will be irrational since E is an odd prime.

VII term in RHS of equation (8), after multiplying by the respective terms and substituting for $\{a(cd)\}$ is

$$= \left(-\sqrt{R^{5/3} t^p} \right) \left(\sqrt{Et^p} + \sqrt{11FE^{1/3}} \right) \left\{ (E^{5/3} R^{1/3}) + (53s^p) + 2\sqrt{E^{5/3} R^{1/3}} \sqrt{53s^p} \right\} (E^{1/3} \sqrt{y}) \\ \times \left(\sqrt{Ex^3} + \sqrt{11E^{1/3} r^p} \right) \left(\sqrt{R^{5/3} y^3} + \sqrt{Fs^p} \right) \left(\sqrt{y^3 r^p} - \sqrt{2347s^p} \right)$$

On multiplying by

$$\left\{ \left(-\sqrt{R^{5/3} t^p} \right) \sqrt{Et^p} (E^{5/3} R^{1/3}) (E^{1/3} \sqrt{y}) \sqrt{11E^{1/3} r^p} \sqrt{R^{5/3} y^3} \sqrt{y^3 r^p} \right\}$$

We get

$$\left\{ -(E^{8/3} R^2 y^3 r^p t^p) \sqrt{11y} \right\}$$

Which is irrational.

VIII term in RHS of equation (8), after multiplying by the respective terms and substituting for $\{a(cd)f\}$ is

$$= (-R) \left(\sqrt{Et^p} + \sqrt{11FE^{1/3}} \right) \left(\sqrt{E^{5/3} R^{1/3}} + \sqrt{53s^p} \right) (E^{1/3} \sqrt{y}) \\ \times \left(\sqrt{Ex^3} + \sqrt{11E^{1/3} r^p} \right) \left(\sqrt{R^{5/3} y^3} + \sqrt{Fs^p} \right) \left(\sqrt{y^3 r^p} - \sqrt{2347s^p} \right) \left(\sqrt{z^3 s^p} - \sqrt{E^{5/3} t^p} \right)$$

On multiplying by

$$= \left\{ (-R) \sqrt{11FE^{1/3}} \sqrt{53s^p} (E^{1/3} \sqrt{y}) \sqrt{Ex^3} \sqrt{Fs^p} (-\sqrt{2347s^p}) \sqrt{z^3 s^p} \right\}$$

$$= \left\{ (FERS^{2p}) \sqrt{53x^3} \sqrt{11y} \sqrt{2347z^3} \right\}$$

Sum of all rational terms on the LHS of equation (8)

$$= \left\{ (FERS^p t^p) \sqrt{53x^3} \sqrt{11y} \sqrt{2347z^3} \right\}$$

Sum of all rational terms on the RHS of equation (8)

$$= \left\{ (FERr^p s^p) \sqrt{53x^3} \sqrt{11y} \sqrt{2347z^3} \right\} \quad \text{(combining I to IV terms)}$$

$$+ \left\{ (53FERr^p s^{2p} t^p) \sqrt{11y} \right\} \quad \text{(vide V term)}$$

$$\begin{aligned}
 & + \left\{ (FERS^{2p}) \sqrt{53x^3} \sqrt{11y} \sqrt{2347z^3} \right\} \quad (\text{vide VI term}) \\
 & = \left\{ (FERS^p) \sqrt{53x^3} \sqrt{11y} \sqrt{2347z^3} \right\} (r^p + s^p) \\
 & \quad + \left\{ (53FEr^p s^{2p} t^p) \sqrt{11y} \right\} \\
 & = \left\{ (FERS^p t^p) \sqrt{53x^3} \sqrt{11y} \sqrt{2347z^3} \right\} \quad (\because r^p + s^p = t^p) \\
 & \quad + \left\{ (53FEr^p s^{2p} t^p) \sqrt{11y} \right\}
 \end{aligned}$$

Equating the rational terms on both sides of equation (8), we get

$$\left\{ (53FEr^p s^{2p} t^p) \sqrt{11y} \right\} = 0$$

Dividing both sides by

$$(53FE) \sqrt{11y}$$

We get

$$(r^p s^{2p} t^p) = 0$$

That is, either $r = 0$; or $s = 0$; or $t = 0$.

This contradicts our hypothesis that all r, s and t are non-zero integers in the equation $r^p + s^p = t^p$, with p any prime > 3 , thus proving that only a trivial solution exists in the equation.

III. CONCLUSION

In this proof, equation (8) is obtained from the two transformation equations for $x^3 + y^3 = z^3$ and $rp + sp = tp$, by substituting the equivalent values of rp, sp , and tp into the equation $r^p + s^p = t^p$. Hence, the result $rst = 0$, which we obtain from equation (8), proves that there are no non-zero integer solutions to the equation $r^p + s^p = t^p$.

DECLARATION STATEMENT

I must verify the accuracy of the following information as the article's author.

- **Conflicts of Interest/Competing Interests:** Based on my understanding, this article does not have any conflicts of interest.
- **Funding Support:** This article has not been funded by any organizations or agencies. This independence ensures that the research is conducted with objectivity and without any external influence.
- **Ethical Approval and Consent to Participate:** The content of this article does not necessitate ethical approval or consent to participate with supporting documentation.
- **Data Access Statement and Material Availability:** The adequate resources of this article are publicly accessible.
- **Author's Contributions:** The authorship of this article is contributed solely.

REFERENCES

1. Hardy G. H. and Wright E. M., *An introduction to the theory of numbers*, 6th ed. Oxford University Press, 2008, pp. 261-586. DOI: <https://doi.org/10.1080/00107510903184414>
2. Lawrence C. Washington, *Elliptic Curves, Number Theory and Cryptography*, 2nd ed., 2003, pp. 445-448. DOI: <https://doi.org/10.1201/9781420071474>
3. Andrew Wiles, *Modular Elliptic Curves and Fermat's Last Theorem*, *Annals of Mathematics*, 1995; 141(3); pp.443-551. DOI: <https://doi.org/10.2307/2118559>
4. 13 Lectures on Fermat's Last Theorem by Paulo Ribenboim, Publisher: Springer, New York, initially published in 1979, p. 159. DOI: <https://doi.org/10.1007/978-1-4684-9342-9>

AUTHOR'S PROFILE



P.N. Seetharaman, B.Sc (Mathematics); B.E (Electrical Engineering), is a retired Executive Engineer from Tamil Nadu Electricity Board. He had served at the Mettur Tunnel Hydro Power Station for ten years and then worked in the Research and Development wing of the Energy Conservation Cell in Chennai. He retired from service in 2002. After retirement, he studied Number Theory, especially Fermat's Last Theorem, and worked on finding and developing an elementary proof for the Theorem.

Disclaimer/Publisher's Note: The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of the Lattice Science Publication (LSP)/ journal and/ or the editor(s). The Lattice Science Publication (LSP)/ journal and/ or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions, or products referred to in the content.