A Comprehensible Proof for Fermat's Last Theorem

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Abstract: Fermat's Last Theorem states that it is impossible to find positive integers A, B and C satisfying the equation $A^n + B^n = C^n$ where n is any integer > 2. Taking the proofs of Fermat for the index n = 4, and Euler for n = 3, it is sufficient to prove the theorem for n = p, any prime > 3. We hypothesise that all r, s, and t are non-zero integers in the equation rp + sp = tp and establish a contradiction. To support the proof in the above equation, we have another equation: $x^3 + y^3 = z^3$. Without loss of generality, we assume that both x and y are non-zero integers, z^3 is a non-zero integer, and z and z^2 are irrational. We transform the above two equations using parameters, incorporating the Ramanujan-Nagell equation. Solving the transformed equations, we prove the theorem.

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I. INTRODUCTION

Around 1637, Pierre de Fermat, the French mathematician, wrote in the margin of a book that the equation An + Bn = Cn has no solution in integers A, B, and C if n is any integer greater than 2. Fermat stated in the margin of the book that he had found a marvellous proof of the theorem, but the margin was too narrow to contain it. His evidence is available only for the index n=4, using the infinite descent method [1].

Many mathematicians, like Sophie Germain and E.E. Kummer, had proved the theorem for particular cases [2]. Number theory has been developed leaps and bounds by the immense contributions of a lot of mathematicians [3]. Finally, after 350 years, the theorem was proved entirely by Prof. Andrew Wiles, using highly complicated mathematical tools and advanced number theory [4].

Here we are trying an elementary proof.

II. ASSUMPTIONS

1) We initially hypothesize that all r, s and t are non-zero integers satisfying the equation

$$r^p + s^p = t^p$$

where p is any prime > 3, with $\gcd(r, s, t) = 1$ and establish a contradiction in this proof. Both s and t cannot simultaneously be squares, that is $\sqrt{st}$ will be irrational.

2) Just to support the proof in the above equation, we have taken another equation.

$$x^3 + y^3 = z^3; \gcd(x, y, z^3) = 1$$

Without loss of generality, we can assume both x and y are non-zero integers, z3 is a non-zero integer, and both z and z2 are irrational. Since we prove the theorem only in the equation $r^p + s^p = t^p$ for all possible integral values of r, s and t we have the choice in having $x=2\times13$; $y=23$; $z^3 = 23^3 + 26^3 = 7^2 \times 607$ or $x = 11$,; $y = 53$; $z^3 = 11^3 + 53^3 = 8^2 \times 2347$ and so on such that their odd prime factors in x, y and z^3 could be chosen as coprimes to each of r, s and t.

In the transformation equation, the pattern and structures are to be maintained, regardless of the odd prime factors of x, y, and z3 used inside the square roots.

3) We have used the Ramanujan-Nagell equation solutions $2^5 = 7 + 5^2$ or $2^7 = 7 + 11^2$ or $2^{15} = 7 + 181^2$ in $2^n = 7 + \ell^2$, where n is odd and $\ell > 1$.

4) Let R be any odd prime, coprime to each of $x, y, z^3, r, s, t, 7$, and ℓ. $F = \ell$ and $E = (xyrst)^{3/11}$

Proof. By random trials, we have created the following equations,

$$\left(\frac{a\sqrt{\ell^{7/3}} + b\sqrt{F^{1/3}}}{\sqrt{2^{3n/2}}}\right)^2 + \left(\frac{c\sqrt{7^{5/3}} + d\sqrt{E^{1/3}}}{\sqrt{\ell^{5/3}}}\right)^2 = \left(\frac{e\sqrt{r} + f\sqrt{F^{5/3}}}{\sqrt{7^{1/3}}}\right)^2$$

and

$$\left(a\sqrt{t} - b\sqrt{53}\right)^2 + \left(\frac{c\sqrt{2^{n/2}} - d\sqrt{R}}{\sqrt{2347}}\right)^2 = \left(e\sqrt{11s} - f\sqrt{E^{5/3}}\right)^2 \quad \ldots \quad (1)$$

to be the transformation equations to $x^3 + y^3 = z^3$ and $r^p + s^p = t^p$, through the parameters called a, b, c, d, e and f. Here $x = 11$, $y = 53$, $z^3 = 11^3 + 53^3 = 8^2 \times 2347$. We have incorporated the Ramanujan-Nagell equation $2^n = 7 + \ell^2$, limited to n as an odd integer and $\ell > 1$. F, E and R be distinct odd primes.

From equation (1), we get

$$a\sqrt{\ell^{7/3}} + b\sqrt{F^{1/3}} = \sqrt{2^{3n/2} x^3} \quad \ldots \quad (2)$$

$$a\sqrt{t} - b\sqrt{53} = \sqrt{r^p} \quad \ldots \quad (3)$$

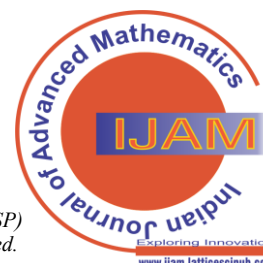$$c\sqrt{7^{5/3}} + d\sqrt{E^{1/3}} = \sqrt{y^3 \ell^{5/3}} \quad \ldots \quad (4)$$

$$c\sqrt{2^{n/2}} - d\sqrt{R} = \sqrt{2347 s^p} \quad \ldots \quad (5)$$

$$e\sqrt{r} + f\sqrt{F^{5/3}} = \sqrt{7^{1/3} z^3} \quad \ldots \quad (6)$$

and

$$e\sqrt{11s} - f\sqrt{E^{5/3}} = \sqrt{t^p} \quad \ldots \quad (7)$$

Solving simultaneously (2) and (3), (4) and (5), (6) and (7), we get

$$a = \left(\sqrt{53 \times 2^{3n/2} x^3} + \sqrt{F^{1/3} r^p}\right)\Big/\left(\sqrt{F^{1/3} t} + \sqrt{53 \times \ell^{7/3}}\right)$$

$$b = \left(\sqrt{2^{3n/2} x^3 t} - \sqrt{\ell^{7/3} r^p}\right)\Big/\left(\sqrt{F^{1/3} t} + \sqrt{53 \times \ell^{7/3}}\right)$$

$$c = \left(\sqrt{R y^3 \ell^{5/3}} + \sqrt{2347 E^{1/3} s^p}\right)\Big/\left(\sqrt{E^{1/3} \times 2^{n/2}} + \sqrt{R \times 7^{5/3}}\right)$$

$$d = \left(\sqrt{2^{n/2} \ell^{5/3} y^3} - \sqrt{2347 \times s^p 7^{5/3}}\right)\Big/\left(\sqrt{E^{1/3} \times 2^{n/2}} + \sqrt{R \times 7^{5/3}}\right) \qquad e = \left(\sqrt{7^{1/3} E^{5/3} z^3} + \sqrt{F^{5/3} t^p}\right)\Big/\left(\sqrt{E^{5/3} r} + \sqrt{11 F^{5/3} s}\right)$$

and
$$f = \left(\sqrt{11 \times 7^{1/3} z^3 s} - \sqrt{r t^p}\right)\Big/\left(\sqrt{E^{5/3} r} + \sqrt{11 \times F^{5/3} s}\right)$$

From (2) & (5), we have

$$\sqrt{2^{n/2}} \times \sqrt{2^{3n/2}} = \left(a\sqrt{\ell^{7/3}} + b\sqrt{F^{1/3}}\right)\left(\sqrt{2347 s^p} + d\sqrt{R}\right)\Big/\left(c\sqrt{x^3}\right)$$

i.e., $\quad 2^n = \left\{(a)\sqrt{2347 s^p \ell^{7/3}} + (ad)\sqrt{R \times \ell^{7/3}} + (b)\sqrt{2347 F^{1/3} s^p} + (bd)\sqrt{F^{1/3} R}\right\}\Big/\left(c\sqrt{x^3}\right)$

From (4) & (6), we get

$$\sqrt{7^{1/3}} \times \sqrt{7^{5/3}} = \left(\sqrt{y^3 \ell^{5/3}} - d\sqrt{E^{1/3}}\right)\left(e\sqrt{r} + f\sqrt{F^{5/3}}\right)\Big/\left(c\sqrt{z^3}\right)$$

i.e., $\quad 7 = \left\{(e)\sqrt{y^3 r \ell^{5/3}} + (f)\sqrt{F^{5/3} y^3 \ell^{5/3}} - (de)\sqrt{E^{1/3} r} - (df)\sqrt{F^{5/3} E^{1/3}}\right\}\Big/\left(c\sqrt{z^3}\right)$

From (2) & (4), we get

$$\sqrt{\ell^{7/3}} \times \sqrt{\ell^{5/3}} = \left(\sqrt{2^{3n/2} x^3} - b\sqrt{F^{1/3}}\right)\left(c\sqrt{7^{5/3}} + d\sqrt{E^{1/3}}\right)\Big/\left(a\sqrt{y^3}\right)$$

i.e., $\quad \ell^2 = \left\{(c)\sqrt{2^{3n/2} \times 7^{5/3} x^3} + (d)\sqrt{E^{1/3} \times 2^{3n/2} x^3} - (bc)\sqrt{F^{1/3} \times 7^{5/3}} - (bd)\sqrt{F^{1/3} E^{1/3}}\right\}\Big/\left(a\sqrt{y^3}\right)$

Substituting the above equivalent values of 2^n, 7, and ℓ^2 in Ramanujan-Nagell equation $2^n = 7 + \ell^2$ after multiplying both sides by $\left\{(ac)\sqrt{x^3 y^3 z^3}\right\}$, we get

$$\left[(a)\sqrt{y^3 z^3}\right]\left\{(a)\sqrt{2347 \times s^p \ell^{7/3}} + (ad)\sqrt{R \ell^{7/3}} + (b)\sqrt{2347 F^{1/3} s^p} + (bd)\sqrt{F^{1/3} R}\right\}$$
$$= \left\{(a)\sqrt{x^3 y^3}\right\}\left\{(e)\sqrt{y^3 r \ell^{5/3}} + (f)\sqrt{y^3 F^{5/3} \ell^{5/3}} - (de)\sqrt{E^{1/3} r} - (df)\sqrt{F^{5/3} E^{1/3}}\right\}$$
$$+ \left\{(c)\sqrt{x^3 z^3}\right\}\left\{(c)\sqrt{2^{3n/2} \times 7^{5/3} x^3} + (d)\sqrt{2^{3n/2} E^{1/3} x^3} - (bc)\sqrt{F^{1/3} \times 7^{5/3}} - (bd)\sqrt{F^{1/3} E^{1/3}}\right\} \quad (8)$$

Our aim is to work out all rational terms in equation (8) after multiplying both sides by

$$\left\{\left(\sqrt{53 \times \ell^{7/3}} + \sqrt{F^{1/3} t}\right)^2 \left(\sqrt{7^{5/3} R} + \sqrt{E^{1/3} \times 2^{n/2}}\right)^2 \left(\sqrt{E^{5/3} r} + \sqrt{11 \times F^{5/3} s}\right)\right\}$$

for freeing from denominators on the parameters *a*, *b*, *c*, *d*, *e* and *f* and again multiplying both sides by
$$\sqrt{2^{n/2} \times 7^{1/3} F^{5/3} E^{5/3} R x t}$$

for getting some rational terms as worked out below, term by term.

I term in LHS of equation (8), after multiplying by the respective terms and substituting for $\{a^2\}$

$$= \sqrt{y^3 z^3}\sqrt{2347 \times s^p \ell^{7/3}}\left(\sqrt{E^{5/3} r} + \sqrt{11 F^{5/3} s}\right)\left\{(7^{5/3} R) + \left(E^{1/3}\sqrt{2^n}\right) + 2\sqrt{7^{5/3} R}\sqrt{2^{n/2} E^{1/3}}\right\}$$
$$\times \sqrt{2^{n/2} \times 7^{1/3} F^{5/3} E^{5/3} R x t}\left\{\left(53 x^3\sqrt{2^{3n}}\right) + \left(F^{1/3} r^p\right) + \left(2\sqrt{53 \times 2^{3n/2} x^3}\sqrt{F^{1/3} r^p}\right)\right\}$$

On multiplying by

$$\left\{\sqrt{y^3 z^3}\sqrt{2347 \times s^p \ell^{7/3}}\sqrt{E^{5/3} r}\left(2\sqrt{7^{5/3} R}\sqrt{2^{n/2} E^{1/3}}\right)\sqrt{2^{n/2} \times 7^{1/3} F^{5/3} E^{5/3} R x t}\left(53 x^3\sqrt{2^{3n}}\right)\right\}$$

We get

$$\left\{2^{2n+1} \times 7 \times 53 R x^3\sqrt{2347 z^3}\sqrt{F^{5/3} \ell^{7/3} E^{11/3} x y^3 r s^p t}\right\}$$

Which is rational since $F = \ell$ and $E^{11/3}(xyrst)$.

II term in LHS of equation (8), after multiplying by the respective terms and substituting for $\{a^2 d\}$, is

$$= \sqrt{y^3 z^3}\sqrt{R \ell^{7/3}}\left(\sqrt{E^{1/3} \times 2^{n/2}} + \sqrt{7^{5/3} R}\right)\left(\sqrt{E^{5/3} r} + \sqrt{11 F^{5/3} s}\right)\sqrt{2^{n/2} \times 7^{1/3} F^{5/3} E^{5/3} R x t}$$
$$\times \left\{\left(53 x^3\sqrt{2^{3n}}\right) + \left(F^{1/3} r^p\right) + 2\sqrt{53 \times 2^{3n/2} x^3}\sqrt{F^{1/3} r^p}\right\}\left(\sqrt{2^{n/2} \ell^{5/3} y^3} - \sqrt{2347 \times s^p 7^{5/3}}\right)$$

On multiplying by

$$\left\{\sqrt{y^3 z^3}\sqrt{R \ell^{7/3}}\sqrt{E^{1/3} \times 2^{n/2}}\sqrt{E^{5/3} r}\sqrt{2^{n/2} \times 7^{1/3} F^{5/3} E^{5/3} R x t}\left(53 x^3\sqrt{2^{3n}}\right)\left(-\sqrt{2347 \times s^p 7^{5/3}}\right)\right\}$$

We get

$$\left\{-\left(2^{2n} \times 7 \times 53 R x^3\right)\sqrt{2347 z^3}\sqrt{F^{5/3} \ell^{7/3} E^{11/3} x y^3 r s^p t}\right\}$$

Which is rational.

III term in LHS of equation (8), after multiplying by the respective terms and substituting for $\{ab\}$

$$= \sqrt{y^3 z^3}\sqrt{2347 \times F^{1/3} s^p}\left(\sqrt{E^{5/3} r} + \sqrt{11 F^{5/3} s}\right)\left\{\left(7^{5/3} R\right) + \left(E^{1/3}\sqrt{2^n}\right) + 2\sqrt{7^{5/3} R}\sqrt{2^{n/2} E^{1/3}}\right\}$$

$$\times \sqrt{2^{n/2} \times 7^{1/3} F^{5/3} E^{5/3} Rxt}\left(\sqrt{53 \times 2^{3n/2} x^3} + \sqrt{F^{1/3} r^p}\right)\left(\sqrt{2^{3n/2} x^3 t} - \sqrt{r^p \ell^{7/3}}\right)$$

On multiplying by

$$\left\{\sqrt{y^3 z^3}\sqrt{2347 \times F^{1/3} s^p}\sqrt{E^{5/3} r}\left(2\sqrt{7^{5/3} R}\sqrt{2^{n/2} E^{1/3}}\right)\sqrt{2^{n/2} \times 7^{1/3} F^{5/3} E^{5/3} Rxt}\sqrt{53 \times 2^{3n/2} x^3}\sqrt{2^{3n/2} x^3 t}\right\}$$

We get

$$\left\{2^{2n+1} \times 7 \times FRx^3 t\sqrt{53 y^3}\sqrt{2347 z^3}\sqrt{E^{11/3} xrs^p}\right\}$$

Which is irrational, since $E^{11/3} = xyrst$, and t is coprime to y, which is 53. Otherwise, we have the choice of assigning an alternative value for y. (see assumptions)

IV term in LHS of equation (8), after multiplying by the respective terms and substituting for $\{(ab)d\}$

$$= \sqrt{F^{1/3} R y^3 z^3}\left(\sqrt{2^{n/2} \times E^{1/3}} + \sqrt{R \times 7^{5/3}}\right)\left(\sqrt{E^{5/3} r} + \sqrt{11 F^{5/3} s}\right)\sqrt{2^{n/2} \times 7^{1/3} F^{5/3} E^{5/3} Rxt}$$

$$\times\left(\sqrt{53 \times 2^{3n/2} x^3} + \sqrt{F^{1/3} r^p}\right)\left(\sqrt{2^{3n/2} x^3 t} - \sqrt{r^p \ell^{7/3}}\right)\left(\sqrt{2^{n/2} \ell^{5/3} y^3} - \sqrt{2347 \times 7^{5/3} s^p}\right)$$

On multiplying by

$$\left\{\sqrt{F^{1/3} R y^3 z^3}\sqrt{2^{n/2} \times E^{1/3}}\sqrt{E^{5/3} r}\sqrt{2^{n/2} \times 7^{1/3} F^{5/3} E^{5/3} Rxt}\sqrt{53 \times 2^{3n/2} x^3}\sqrt{2^{3n/2} x^3 t}\left(-\sqrt{2347 \times 7^{5/3} s^p}\right)\right\}$$

We get

$$\left\{-\left(2^{2n} \times 7 FRx^3 t\right)\sqrt{53 y^3}\sqrt{2347 z^3}\sqrt{E^{11/3} xrs^p}\right\}$$

Which is irrational.

I term in RHS of equation (8), after multiplying by the respective terms and substituting for $\{ae\}$

$$= \left(y^3\sqrt{rx^3 \ell^{5/3}}\right)\left(\sqrt{F^{1/3} t} + \sqrt{53 \times \ell^{7/3}}\right)\left(\sqrt{53 \times 2^{3n/2} x^3} + \sqrt{F^{1/3} r^p}\right)\sqrt{2^{n/2} \times 7^{1/3} F^{5/3} E^{5/3} Rxt}$$

$$\times\left\{\left(E^{1/3}\sqrt{2^n}\right) + \left(7^{5/3} R\right) + 2\sqrt{7^{5/3} R}\sqrt{2^{n/2} E^{1/3}}\right\}\left(\sqrt{7^{1/3} E^{5/3 \cdot} z^3} + \sqrt{F^{5/3} t^p}\right)$$

On multiplying by

$$\left\{\left(y^3\sqrt{rx^3 \ell^{5/3}}\right)\sqrt{53 \times \ell^{7/3}}\sqrt{F^{1/3} r^p}\sqrt{2^{n/2} \times 7^{1/3} F^{5/3} E^{5/3} Rxt}\left(2\sqrt{7^{5/3} R}\sqrt{2^{n/2} E^{1/3}}\right)\sqrt{F^{5/3} t^p}\right\}$$

We get

$$\left\{\left(2 \times 7 ER\ell^2 x^2 y^3\right)\sqrt{(rt)^{p+1}}\sqrt{53 \times F^{11/3} 2^n}\right\}$$

Which is irrational.

II term in RHS of equation (8), after multiplying by the respective terms and substituting for $\{af\}$

$$= \left(y^3\sqrt{F^{5/3} x^3 \ell^{5/3}}\right)\left(\sqrt{F^{1/3} t} + \sqrt{53 \times \ell^{7/3}}\right)\left(\sqrt{53 \times 2^{3n/2} x^3} + \sqrt{F^{1/3} r^p}\right)\sqrt{2^{n/2} \times 7^{1/3} F^{5/3} E^{5/3} Rxt}$$

$$\left\{\left(7^{5/3} R\right) + \left(E^{1/3}\sqrt{2^n}\right) + 2\sqrt{7^{5/3} R}\sqrt{2^{n/2} E^{1/3}}\right\}\left(\sqrt{11 \times 7^{1/3} z^3 s} - \sqrt{rt^p}\right)$$

On multiplying by

$$\left\{\left(y^3\sqrt{F^{5/3} x^3 \ell^{5/3}}\right)\sqrt{53 \times \ell^{7/3}}\sqrt{F^{1/3} r^p}\sqrt{2^{n/2} \times 7^{1/3} F^{5/3} E^{5/3} Rxt}\left(2\sqrt{7^{5/3} R}\sqrt{2^{n/2} E^{1/3}}\right)\left(-\sqrt{rt^p}\right)\right\}$$

We get

$$\left\{-\left(2 \times 7 ER\ell^2 x^3 y^3\right)\sqrt{(rt)^{p+1}}\sqrt{53 \times F^{11/3} 2^n}\right\}$$

Which is irrational.

III term in RHS of equation (8), after multiplying by the respective terms and substituting for $\{ade\}$

$$= \left(-\sqrt{E^{1/3} x^3 y^3 r}\right)\left(\sqrt{F^{1/3} t} + \sqrt{53 \times \ell^{7/3}}\right)\left(\sqrt{E^{1/3} \times 2^{n/2}} + \sqrt{R \times 7^{5/3}}\right)\sqrt{2^{n/2} \times 7^{1/3} F^{5/3} E^{5/3} Rxt}$$

$$\times\left\{\sqrt{53 \times 2^{3n/2} x^3} + \sqrt{F^{1/3} r^p}\right\}\left(\sqrt{2^{n/2} \ell^{5/3} y^3} - \sqrt{2347 \times 7^{5/3} s^p}\right)\left(\sqrt{7^{1/3} E^{5/3} z^3} + \sqrt{F^{5/3} t^p}\right)$$

On multiplying by

$$\left\{\left(-\sqrt{E^{1/3}x^3y^3r}\right)\sqrt{53\times\ell^{7/3}}\sqrt{R\times7^{5/3}}\sqrt{2^{n/2}\times7^{1/3}F^{5/3}E^{5/3}Rxt}\sqrt{53\times2^{3n/2}x^3}\left(-\sqrt{2347\times7^{5/3}s^p}\right)\sqrt{7^{1/3}E^{5/3}z^3}\right\}$$ We get

$$\left\{\left(2^n\times7^2\times53Rx^3\right)\sqrt{2347z^3}\sqrt{F^{5/3}\ell^{7/3}E^{11/3}xy^3rs^pt}\right\}$$

Which is rational, since $z^3=8^2\times2347$; $F=\ell$ and $E^{11/3}=(xyrst)$.

IV term in RHS of equation (8), after multiplying by the respective terms and substituting for $\{(adf\}$

$$=\left(-\sqrt{x^3y^3}\right)\sqrt{F^{5/3}E^{1/3}}\left(\sqrt{F^{1/3}t}+\sqrt{53\times\ell^{7/3}}\right)\left(\sqrt{E^{1/3}\times2^{n/2}}+\sqrt{R\times7^{5/3}}\right)\sqrt{2^{n/2}\times7^{1/3}F^{5/3}E^{5/3}Rxt}$$

$$\times\left\{\sqrt{53\times2^{3n/2}x^3}+\sqrt{F^{1/3}r^p}\right\}\left(\sqrt{2^{n/2}\ell^{5/3}y^3}-\sqrt{2347\times7^{5/3}s^p}\right)\left(\sqrt{11\times7^{1/3}z^3s}-\sqrt{rt^p}\right)$$

On multiplying by

$$\left\{\left(-\sqrt{x^3y^3}\right)\sqrt{F^{5/3}E^{1/3}}\sqrt{53\times\ell^{7/3}}\sqrt{R\times7^{5/3}}\sqrt{2^{n/2}\times7^{1/3}F^{5/3}E^{5/3}Rxt}\sqrt{53\times2^{3n/2}x^3}\left(-\sqrt{2347\times7^{5/3}s^p}\right)\sqrt{11\times7^{1/3}z^3s}\right\}$$

We get

$$\left\{\left(2^n\times7^2\times53ERx^3\right)\sqrt{11x}\sqrt{s^{p+1}}\sqrt{F^{10/3}\ell^{7/3}ty^3}\sqrt{2347z^3}\right\}$$

Which will be irrational since $F=\ell$.

V term in RHS of equation (8), after multiplying by the respective terms and substituting for $\{c^2\}$

$$=\left(x^3\right)\sqrt{2^{3n/2}\times7^{5/3}z^3}\left\{\left(F^{1/3}t\right)+\left(53\ell^{7/3}\right)+2\sqrt{53\ell^{7/3}}\sqrt{F^{1/3}t}\right\}\sqrt{2^{n/2}\times7^{1/3}F^{5/3}E^{5/3}Rxt}$$

$$\times\left(\sqrt{E^{5/3}r}+\sqrt{11F^{5/3}s}\right)\left\{\left(Ry^3\ell^{5/3}\right)+\left(2347E^{1/3}s^p\right)+2\sqrt{2347E^{1/3}s^p}\sqrt{Ry^3\ell^{5/3}}\right\}$$

On multiplying by

$$\left(x^3\right)\sqrt{2^{3n/2}\times7^{5/3}z^3}\left(53\ell^2\sqrt{\ell^{2/3}}\right)\sqrt{2^{n/2}\times7^{1/3}F^{5/3}E^{5/3}Rxt}\sqrt{E^{5/3}r}\left(2\sqrt{2347E^{1/3}s^p}\sqrt{Ry^3\ell^{5/3}}\right)$$

We get

$$\left\{\left(2^{n+1}\times7\times53Rx^3\ell^2\right)\sqrt{2347z^3}\sqrt{F^{5/3}\ell^{7/3}E^{11/3}xy^3rs^pt}\right\}$$

Which is rational.

VI term in RHS of equation (8), after multiplying by the respective terms and substituting for $\{cd\}$ is

$$=\left(x^3\right)\sqrt{2^{3n/2}E^{1/3}z^3}\left\{\left(F^{1/3}t\right)+\left(53\times\ell^{7/3}\right)+\left(2\sqrt{53F^{1/3}\ell^{7/3}t}\right)\right\}\left(\sqrt{E^{5/3}r}+\sqrt{11F^{5/3}s}\right)$$

$$\times\sqrt{2^{n/2}\times7^{1/3}F^{5/3}E^{5/3}Rxt}\left(\sqrt{R\ell^{5/3}y^3}+\sqrt{2347E^{1/3}s^p}\right)\left(\sqrt{2^{n/2}\ell^{5/3}y^3}-\sqrt{2347\times7^{5/3}s^p}\right)$$

On multiplying by

$$\left\{\left(x^3\right)\sqrt{2^{3n/2}E^{1/3}z^3}\left(53\ell^2\sqrt{\ell^{2/3}}\right)\sqrt{E^{5/3}r}\sqrt{2^{n/2}\times7^{1/3}F^{5/3}E^{5/3}Rxt}\sqrt{R\ell^{5/3}y^3}\left(-\sqrt{2347\times7^{5/3}s^p}\right)\right\}$$

We get

$$\left\{-\left(2^n\times7\times53Rx^3\ell^2\right)\sqrt{2347z^3}\sqrt{F^{5/3}\ell^{7/3}E^{11/3}xy^3rs^pt}\right\}$$

Which is rational.

VII term in RHS of equation (8), after multiplying by the respective terms and substituting for $\{bc^2\}$, is

$$=\left(-\sqrt{7^{5/3}F^{1/3}x^3z^3}\right)\left(\sqrt{F^{1/3}t}+\sqrt{53\times\ell^{7/3}}\right)\left(\sqrt{E^{5/3}r}+\sqrt{11F^{5/3}s}\right)\sqrt{2^{n/2}\times7^{1/3}F^{5/3}E^{5/3}Rxt}$$

$$\times\left(\sqrt{2^{3n/2}x^3t}-\sqrt{r^p\ell^{7/3}}\right)\left\{\left(Ry^3\ell^{5/3}\right)+\left(2347E^{1/3}s^p\right)+\left(2\sqrt{2347E^{1/3}s^p}\right)\sqrt{R\ell^{5/3}y^3}\right\}$$

On multiplying by

$$\left\{\left(-\sqrt{7^{5/3}F^{1/3}x^3z^3}\right)\sqrt{F^{1/3}t}\sqrt{E^{5/3}r}\sqrt{2^{n/2}\times7^{1/3}F^{5/3}E^{5/3}Rxt}\sqrt{2^{3n/2}x^3t}\left(\left(2\sqrt{2347E^{1/3}s^p}\right)\sqrt{R\ell^{5/3}y^3}\right)\right\}$$

We get

$$\left\{-\left(2^{n+1}\times7\times FRx^3t\right)\sqrt{2347z^3}\sqrt{F^{1/3}\ell^{5/3}E^{11/3}xy^3rs^pt}\right\}$$

Which is rational.

VIII term in RHS of equation (8), after multiplying by the respective terms and substituting for $\{b(cd)\}$ is

$$=\left(-\sqrt{F^{1/3}E^{1/3}x^3z^3}\right)\left(\sqrt{F^{1/3}t}+\sqrt{53\times\ell^{7/3}}\right)\left(\sqrt{E^{5/3}r}+\sqrt{11F^{5/3}s}\right)\sqrt{2^{n/2}\times7^{1/3}F^{5/3}E^{5/3}Rxt}$$

$$\times\left(\sqrt{2^{3n/2}x^3t}-\sqrt{r^p\ell^{7/3}}\right)\left(\sqrt{R\ell^{5/3}y^3}+\sqrt{2347E^{1/3}s^p}\right)\left(\sqrt{2^{n/2}\ell^{5/3}y^3}-\sqrt{2347\times7^{5/3}s^p}\right)$$

On multiplying by

$$\left\{\left(-\sqrt{F^{1/3}E^{1/3}x^3z^3}\right)\sqrt{F^{1/3}t}\sqrt{E^{5/3}r}\sqrt{2^{n/2}\times 7^{1/3}F^{5/3}E^{5/3}Rxt}\sqrt{2^{3n/2}x^3t}\sqrt{R\ell^{5/3}y^3}\left(-\sqrt{2347\times 7^{5/3}s^p}\right)\right\}$$

We get

$$\left\{\left(2^n\times 7\times FRx^3t\right)\sqrt{2347z^3}\sqrt{F^{1/3}\ell^{5/3}E^{11/3}xy^3rs^pt}\right\}$$

Which is rational

Sum of all rational terms in the LHS of equation (8)

$$=\left\{\left(2^{2n}\times 7\times 53Rx^3\right)\sqrt{2347z^3}\sqrt{F^{5/3}\ell^{7/3}E^{11/3}xy^3rs^pt}\right\}\quad\text{(combining I \& II terms)}$$

Sum of all rational terms in the RHS of equation (8) is

$$=\left\{\left(2^n\times 7^2\times 53Rx^3\right)\sqrt{2347z^3}\sqrt{F^{5/3}\ell^{7/3}E^{11/3}xy^3rs^pt}\right\}\quad\text{(vide III term)}$$

$$+\left\{\left(2^n\times 7\times 53Rx^3\ell^2\right)\sqrt{2347z^3}\sqrt{F^{5/3}\ell^{7/3}E^{11/3}xy^3rs^pt}\right\}\quad\text{(combining V \& VI terms)}$$

$$-\left\{\left(2^n\times 7\times FRx^3t\right)\sqrt{2347z^3}\sqrt{F^{1/3}\ell^{5/3}E^{11/3}xy^3rs^pt}\right\}\quad\text{(combining VII \& VIII terms)}$$

$$=\left\{\left(2^{2n}\times 7\times 53Rx^3\ell^2\right)\sqrt{2347z^3}\sqrt{F^{5/3}\ell^{7/3}E^{11/3}xy^3rs^pt}\right\}\quad\left(\because\ 7+\ell^2=2^n\right)$$

$$-\left\{\left(2^n\times 7\times FRx^3t\right)\sqrt{2347z^3}\sqrt{F^{1/3}\ell^{5/3}E^{11/3}xy^3rs^pt}\right\}$$

Equating the rational terms on both sides of equation (8), we get

$$-\left(2^n\times 7\times FRx^3t\right)\sqrt{2347z^3}\sqrt{F^{1/3}\ell^{5/3}E^{11/3}xy^3rs^pt}=0$$

Dividing both sides by

$$\left\{-\left(2^n\times 7\times FRx^3\right)\sqrt{2347z^3}\right\}$$

We get

$$\left(t\sqrt{F^{1/3}\ell^{5/3}E^{11/3}xy^3rs^pt}\right)=0$$

i.e., either $r=0$ or $s=0$ or $t=0$.

This contradicts our hypothesis that all r, s and t are non-zero integers in the equation $r^p+s^p=t^p$, with p any prime >3, thus proving that only a trivial solution exists in the equation.

III. CONCLUSION

Since equation (8) in this proof has been derived directly from the transformation equations the result that we have obtained on equating the rational terms on both sides of equation (8) should reflect on the Fermat's Equation $r^p+s^p=t^p$, thus proving that only a trivial solution exists in the equation $r^p+s^p=t^p$.

The central hypothesis that we made in this proof, namely that r, s, and t are non-zero integers, has been shattered by the result $rst=0$, thus proving the theorem.

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I must verify the accuracy of the following information as the article's author.

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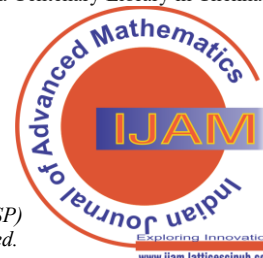- **Author's Contributions:** The authorship of this article is contributed solely.

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