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Pseudo Number 0.999...

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***Abstract:** As we know that the number 0.999... is a rational number and mathematicians, teachers and students of mathematics generally wonder whether the number 0.999... and the number 1 are really equal or not. This problem is one of the ancient problems of mathematics and is generally the most asked problem. While trying to solve the above problem, a new fact is found that the number 0.999... is not a real number and with this a satisfactory solution of the above problem is obtained. In this paper, we do analysis about the existence of the number 0.999... on the real number line and prove that the number 0.999... is only a virtual number.*

***Keywords:** Number System, Mathematical Reasoning.*

I. INTRODUCTION

A number r, which can be written in the form $^a/b$, where a and b are co-primes and b doesn't equal to zero, is called a rational number. On division of a by b, two main situations create - In first situation, the remainder becomes zero and r has a terminating decimal expansion. For some examples, $^1/_4 = 0.25$, $^1/_5 = 0.2$

And $^1/_{10} = 0.1$ etc. And this situation creates because the prime factorisation of b is of the form $2^m \times 5^n$, where m and n are whole numbers. And the converse is also true.

In second situation, the remainder never becomes zero and r has a non-terminating repeating decimal expansion. For some examples, $^1/_3 = 0.333...$,

$^2/_9 = 0.222...$ etc. And this situation creates because the prime factorisation of b is not of the form $2^m \times 5^n$, where m and n are whole numbers [1], [3]. But the converse is not true because 0.999... is equal to 1. But if number 0.999... does not exist on the real number line, then the converse is also true.

As we know that all real numbers are made up of 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9.

0/9 = 0.000... = 0, 1/9 = 0.111... , 2/9 = 0.222... , 3/9 = 0.333... ,

4/9 = 0.444... , 5/9 = 0.555... , 6/9 = 0.666... , 7/9 = 0.777...

And 8/9 = 0.888...

But 9/9 = 1.

Hence it appears from the above that the number 0.999... does not exist.

But if we study the following geometric series with infinite the number of terms, 0.9 + 0.09 + 0.009 + ... = 0.999...

Where a = 0.9 and r = 0.1 [5].

So the number 0.999... appears to exist.

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II. MATERIALS AND METHODS

First we make a statement q.

q: The number 0.999... exists on the real number line.

We use proof by contradiction to prove statement q to be false [4].

Theorem: 1

The number 0.999... does not exist on the real number line rather, it is only a virtual number.

Proof: Let statement q be a true statement. We know that

$0.9 = 9/10 = (10 -1)/10 = 1 – (1/10) = 1 – 0.1 = 1 – (0.1)^1$,

$0.99 = 99/100 = (100-1)/100 = 1 – (1/100) = 1 – 0.01 = 1 – (0.1)^2$,

$0.999 = 999/1000 = (1000-1)/1000 = 1 – (1/1000) = 1 – 0.001 = 1 – (0.1)^3$,

Similarly,

If the number $0.999...9_{(n\ –\ times)}$ consists of n numbers of 9 after the decimal point and where n is a natural number, then

$0.999...9_{(n\ –\ times)} = 1 – (0.1)^n$.

Hence it is clear from the above that $0.999... = 1 – (0.1)^\infty$.

Just as solving $(0.1)^n$ gives n–1 number of zeros after the decimal and 1 is obtained at the nth place after the decimal, similarly solving $(0.1)^\infty$ gives the number of zeros after the decimal $\infty–1 = \infty$ means infinity, which means that solving $(0.1)^\infty$ always gives zero after the decimal because the process of getting zero after the decimal is not the end rather, the process of getting zero goes on and 1 is never found, so solving for $(0.1)^\infty$ gives a result equal to zero.

That is, $(0.1)^\infty = 0.000... = 0$.

So $0.999... = 1 – (0.1)^\infty$

$0.999... = 1 – 0$

$0.999... = 1$.

Since, Each natural number is divisible by 1 So, 1 is divisible by 1 and we can write $^1/_1 = 1$. Then, we get $^1/_1 = 0.999...$.

Which is possible only if the number 9 is divisible by the number 1 but numbers 1 and 10 are not divisible by the number 1. In this situation, if the number 1 is divided by the number 1, we get $^1/_1 = 0.999...$.

From the above result it is known that the number 9 is divisible by 1 rather, numbers 1 and 10 are not divisible by 1.

Which is a contradiction because each natural number is divisible by 1, therefore 1 and 10 are also divisible by 1.

This contradiction arises because we accept statement q as a true statement. Hence statement q is a false statement.

So, the number 0.999... does not exist on the real number line and it is only a virtual number.

Once again let statement q be a true statement but 1 is greater than 0.999... .

From the above, 1 > 0.999...

As we know that $^{1}/_{9} = 0.111...$
$^{1}/_{9} = 0.1 + 0.01 + 0.001 + ...$
It is a geometric series of infinite terms
with a = 0.1 and r = 0.1 [5],
$^{1}/_{9} = [\ 0.1 \times \{\ 1 - (\ 0.1\ )^{\infty}\ \}\ ] \div (\ 1 - 0.1\ )$
$^{1}/_{9} = (\ 0.1 \times 0.999...\ ) \div 0.9$
$^{1}/_{9} = {}^{1}/_{9} \times 0.999...$
$1 = 0.999...$
We start from 1 > 0.999... but we get 1 = 0.999... .
Which is a contradiction. And this contradiction arises because we accept statement q as a true statement.
Hence statement q is a false statement.
Now consider another situation.
We know that 1 < 2. But if we assume that $^{0}/_{0} = 1$, then we get 1 = 2 [2]. This is a contradiction to be resolved by accepting that $^{0}/_{0}$ is never equal to 1. Similarly, in the above situation also we accept that the number 0.999... is not a real number. so, the number 0.999... does not exist on the real number line and it is only a virtual number.
As we know that while proving the theorem, some such facts are obtained which can be easily understood, called sub-theorems. While proving the above theorem, the following two sub-theorems are obtained: Sub-theorem: (1) The number 0.999... and the number 1 are not equal.
And this sub-theorem is the answer to the above most asked problem. As we know that statement p is a true statement.
p : Let r = $^{a}/_{b}$, be a rational number such that a and b are co-primes and prime factorisation of b is not of the form $2^m \times 5^n$, where m and n are whole numbers then r has a non-terminating repeating decimal expansion.
Sub-theorem: (2)
The converse of statement p is also true.

III. RESULT

The number 0.999... is only a virtual number, not a real number and does not exist on the real number line. Hence the number 0.999... is called a pseudo number.

IV. CONCLUSION

In this paper, we first consider two situations from which we get an estimate of whether or not the number 0.999... exists. And then we prove by using "Proof By Contradiction" that there doesn't exist a single point on the real number line that represents the number 0.999... .
And with this we get the following two sub-theorems:
(1) The number 0.999... and the number 1 are not equal.
(2) The converse of statement p is also true.

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