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On the Results of Coffy and Moli

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Abstract*:. *The primary aim of this article is Generalization leading to search of integrals of the type* $\int \frac{logP}{logQ}dx = \int FG^n dx$ *where both P, Q are functions of x only and also verification of results by using new trend. In this direction of verification, converting integrals into ordinary differential equations and finding the solutions, has significant trend and fresh impetus in the recent years. In this paper, along with the generalization of integrals, formation of ordinary differential equations and their solutions were also listed. Here in this article we obtained significant differential equations based on integrals of Russell [1], type. In the first part, generalizations of preliminary lemmas along with new integrals of logarithmic integrand were evaluated on the lines Mark Coffy [4]. Then first order linear differential equations were obtained for such integrals, to verify the truth of the solutions.

Keywords*: *Beta-functions, Definite Integrals, First order linear Differential Equations. Gamma-functions, Taylor Series, Ordinary Differential Equations.

I. INTRODUCTION

In recent years applications of integrals and generalizations, has took a significant new trend, and different approaches since from its inception in the research papers of Mark Coffy and Moli [3],[4]. These researches have included many facts connected with convergence of Series, Hyperbolic Functions, Beta Gamma functions[7] and many more leading to self –reciprocal Fourier Transformers[7,8]

In their results Russell style of integrals, were presented. But such Russell style of integrals initiated new fresh impetus in multiple directions and applications of definite integrals, creating new trend and interesting results in recent years.

Creation of new trend and significant results concerning with differential equations, is the prime focus of this research article. In this article we presented the results, as the initial approach with another intension of unveiling pros and cons of Russell style integrals. The idea of Russell type integrals involves representing F(x) as $\int PQ^n dx$. The infinite series of integrand is obtained, then the summation and integration are interchanged to obtain integral value in terms of sum of series. Then the series so obtained converge to known functions. Earlier authors mentioned that the study of such type of integrals have many applications in the calculation of hyper volumes, Feynman diagrams[9], and have the origin in the research article "On the theory of Definite Integrals ", by W H L Russell [1][2] leads to several significant researches. It took fresh impetus gradually, by various authors. Among these, T Amdeberhan and V H Moli [3] pointed out dozen integrals, which were prime focus of many researchers. Here we are not targeted to solve any specific problems, taking ideas from survey of earlier research papers, but focused on providing a subject of new investigations. My efforts to pursue integrals by constructing differential equations [5] [6] using Leibnitz rule [10] and then solving these differential equations by different techniques were successful. The results listed in this paper; at the outset has simple consequences of earlier results, but in the latter part new results were incorporated.

Lemma 1: [4] $\int_0^\pi \theta sin\theta(cos\theta)^{2n}d\theta = \frac{\pi}{2n+1}$

Proof: $I_{2n} = \int_0^\pi \theta sin\theta(cos\theta)^{2n}d\theta$

$$= \int_0^\pi (\pi-\theta)sin\theta(cos\theta)^{2n}d\theta$$

$$= \pi\int_0^\pi sin\theta(cos\theta)^{2n}d\theta - I_{2n}$$

$$=> 2I_{2n} = \pi\int_0^\pi sin\theta(cos\theta)^{2n}d\theta$$

$$I_{2n} = \pi\left[\frac{-(cos\theta)^{2n+1}}{2n+1}\right]_0^{\pi/2} = \frac{\pi}{2n+1} \quad \ldots(1.1)$$

Lemma 2 [4]: $\int_0^\pi \frac{\theta sin\theta}{1-x^2cos^2\theta}d\theta = \frac{\pi}{x}tanh^{-1}x$

Proof: $\int_0^\pi \frac{\theta sin\theta}{1-x^2cos^2\theta}d\theta =$

$$\int_0^\pi \theta sin\theta\sum_{n=0}^{\infty} x^{2n}(cos\theta)^{2n}d\theta$$

$$=\sum_{n=0}^{\infty} x^{2n}\int_0^\pi \theta sin\theta(cos\theta)^{2n}d\theta \quad \ldots(1.2)$$

Substituting (1.1) in (1.2), one can obtain

$$=\sum_{n=0}^{\infty} x^{2n}\frac{\pi}{(2n+1)} = \frac{\pi}{x}\sum_{n=0}^{\infty}\frac{x^{2n+1}}{(2n+1)}$$

$$=\frac{\pi}{x}tanh^{-1}x$$

Thus we see that, $\int_0^\pi \frac{\theta sin\theta d\theta}{1-x^2cos^2\theta}$

$$=\sum_{n=0}^{\infty}\frac{x^{2n}\pi}{2n+1} = \frac{\pi}{x}\sum_{n=0}^{\infty}\frac{x^{2n+1}}{2n+1} = \frac{\pi}{x}tanh^{-1}x = \frac{\pi}{x}log\left(\frac{1+x}{1-x}\right)$$

Lemma 3: $\int_0^\pi \frac{\theta^2 sin\theta d\theta}{1-x^2cos^2\theta} =$

$$\sum_{n=0}^{\infty}\frac{x^{2n}}{(2n+1)}\left(2\int_0^\pi \theta sin\theta(cos^{2n+1}(\theta))d\theta - \pi^2\right)$$

Proof: $\int_0^\pi \frac{\theta^2 sin\theta d\theta}{1-x^2cos^2\theta} = \sum_{n=0}^{\infty} x^{2n}\int_0^\pi \theta^2 sin\theta(cos^{2n}(\theta))d\theta$

Integrating by parts, taking u=$\theta^2(cos^{2n}(\theta))$ and v= $sin\theta$,

we get, $\sum_{n=0}^{\infty}\frac{x^{2n}}{(2n+1)}\left(2\int_0^\pi \theta sin\theta(cos^{2n+1}(\theta))d\theta - \pi^2\right) = -\pi^2\sum_{n=0}^{\infty}\frac{x^{2n}}{(2n+1)} +$

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$$\left(2\sum_{n=0}^{\infty}\frac{x^{2n}}{(2n+1)}\int_0^{\pi}\theta\sin\theta(\cos^{2n+1}(\theta))d\theta\right) \qquad \ldots(1.3)$$

In equation (1.3) the first part $-\pi^2\sum_{n=0}^{\infty}\frac{x^{2n}}{(2n+1)}$

$$=\frac{-\pi^2}{x}\sum_{n=0}^{\infty}\frac{x^{2n+1}}{(2n+1)}=\frac{-\pi^2}{x}\tanh^{-1}x \qquad \ldots(1.4)$$

and for the second part of (1.3), consider I=$\int_0^{\pi}\theta\sin\theta(\cos^{2n+1}(\theta))d\theta$ integrating by parts gives the following result.

$$I=\int_0^{\pi}\theta\sin\theta(\cos^{2n+1}(\theta))d\theta$$

$$=[t\cos^{2n+1}t(-\cos t)]_0^{\pi}+\int_0^{\pi}\cos t[\cos^{2n+1}t+(2n+1)t\cos^{2n}t(-\sin t)]dt$$

$$=\pi+\int_0^{\pi}\cos^{2n+2}t\,dt-\int_0^{\pi}(2n+1)t\cos^{2n}t(\sin t)dt$$

Hence $\int_0^{\pi}t\cos^{2n}t(\sin t)dt$

$$=\frac{1}{2(n+1)}\left\{\pi+\int_0^{\pi}\cos^{2n+2}t\,dt\right\}$$

$$=\frac{\pi}{2(n+1)}+\frac{\pi}{2(n+1)}\frac{\beta(\frac{1}{2},\frac{2n+1}{2})((-1)^{2n}+1)}{2} \qquad \ldots(1.5)$$

Using (1.4) and (1.5) in (1.3), we note that

$$\int_0^{\pi}\frac{\theta^2\sin\theta d\theta}{1-x^2\cos^2\theta}=\frac{-\pi^2}{x}\tanh^{-1}x+\sum_{n=0}^{\infty}\frac{x^{2n}}{2n+1}\left[\frac{\pi}{2(n+1)}+\frac{\pi}{2(n+1)}\frac{\beta(\frac{1}{2},\frac{2n+1}{2})((-1)^{2n}+1)}{2}\right]$$

$$=\frac{-\pi^2}{x}\tanh^{-1}x+\frac{\pi}{2x}\sum_{n=0}^{\infty}\frac{x^{2n+1}}{(2n+1)(n+1)}+\frac{\pi}{2x}\sum_{n=0}^{\infty}\frac{x^{2n+1}}{(2n+1)(n+1)}\frac{\beta(\frac{1}{2},\frac{2n+1}{2})((-1)^{2n}+1)}{2}$$

$$=\frac{-\pi^2}{x}\tanh^{-1}x+\frac{\pi}{2x}\sum_{n=0}^{\infty}x^{2n+1}\left[\frac{2}{(2n+1)}-\frac{1}{(n+1)}\right]+\frac{\pi}{2x}\sum_{n=0}^{\infty}x^{2n+1}\left[\frac{2}{(2n+1)}-\frac{1}{(n+1)}\right]\frac{\beta(\frac{1}{2},\frac{2n+1}{2})((-1)^{2n}+1)}{2}$$

$$=\frac{\pi-\pi^2}{x}\tanh^{-1}x-\frac{\pi}{2x}\sum_{n=0}^{\infty}\left[\frac{x^{2n+1}}{(n+1)}\right]+\frac{\pi}{2x}\sum_{n=0}^{\infty}x^{2n+1}\left[\frac{2}{(2n+1)}-\frac{1}{(n+1)}\right]\frac{\beta(\frac{1}{2},\frac{2n+1}{2})((-1)^{2n}+1)}{2}$$

$$=\left(\frac{\pi-\pi^2}{x}+\frac{\pi}{2x}\beta(\frac{1}{2},\frac{2n+1}{2})((-1)^{2n}+1)\right)\tanh^{-1}x-\frac{\pi}{2x}\sum_{n=0}^{\infty}\left[\frac{x^{2n+1}}{(n+1)}\right]\left(1+\frac{\beta(\frac{1}{2},\frac{2n+1}{2})((-1)^{2n}+1)}{2}\right)$$

We set a distinct result here to give rise to another view of generalization

Lemma 4 . $\int_0^{\pi}\frac{(1-x\cos\theta)\theta\sin\theta d\theta}{(1+x\cos\theta)}$

$$=\frac{2}{\pi}\left\{\int_0^{\pi}\frac{\theta\sin\theta d\theta}{1-x^2\cos^2\theta}+x^2\int_0^{\pi}\frac{\theta\cos^2\theta\sin\theta d\theta}{1-x^2\cos^2\theta}\right\}$$

$$=\frac{4}{x}\left(\tanh^{-1}x-\frac{1}{2}\right)$$

Proof: $I(x)=\int_0^{\pi}\frac{(1+x\cos\theta)\theta\sin\theta d\theta}{(1-x\cos\theta)}$

$$=\int_0^{\pi}\frac{(1+x\cos(\pi-\theta))(\pi-\theta)\sin(\pi-\theta)d\theta}{(1-x\cos(\pi-\theta))}$$

$$=\int_0^{\pi}\frac{(1-x\cos\theta)(\pi-\theta)\sin\theta d\theta}{(1+x\cos\theta)}$$

$$=\pi\int_0^{\pi}\frac{(1-x\cos\theta)\sin\theta d\theta}{(1+x\cos\theta)}-\int_0^{\pi}\frac{(1-x\cos\theta)\theta\sin\theta d\theta}{(1+x\cos\theta)}$$

$$=>\pi\int_0^{\pi}\frac{(1-x\cos\theta)\sin\theta d\theta}{(1+x\cos\theta)}=\int_0^{\pi}\left(\frac{(1+x\cos\theta)}{(1-x\cos\theta)}+\frac{(1-x\cos\theta)}{(1+x\cos\theta)}\right)\theta\sin\theta d\theta$$

$$=\int_0^{\pi}\left[\frac{(1+x\cos\theta)^2+(1-x\cos\theta)^2}{(1-x^2\cos^2\theta)}\right]\theta\sin\theta d\theta$$

$$=\int_0^{\pi}\left[\frac{(1+x\cos\theta)^2+(1-x\cos\theta)^2}{(1-x^2\cos^2\theta)}\right]\theta\sin\theta d\theta$$

$$=\int_0^{\pi}\left[\frac{2(1+(x\cos\theta)^2)}{(1-x^2\cos^2\theta)}\right]\theta\sin\theta d\theta$$

$$=2\int_0^{\pi}\left[\frac{1}{(1-x^2\cos^2\theta)}+\frac{x^2\cos^2\theta}{(1-x^2\cos^2\theta)}\right]\theta\sin\theta d\theta$$

$$=2\left\{\frac{\pi}{x}\tanh^{-1}x+\sum_{n=0}^{\infty}\frac{\pi x^{\overset{2}{n+1}}}{2n+3}\right\}$$

$$=2\left\{\frac{\pi}{x}\tanh^{-1}x+\frac{\pi}{x}\sum_{n=0}^{\infty}\frac{x^{2(n+1)}}{2(n+1)+1}\right\}$$

$$=2\left\{\frac{\pi}{x}\tanh^{-1}x+\frac{\pi}{x}(\tanh^{-1}x-1)\right\}$$

$$=\frac{4\pi}{x}\left(\tanh^{-1}x-\frac{1}{2}\right)$$

Theorem 1: $\int_0^{\pi}\frac{\theta^3\sin\theta d\theta}{1-x^2\cos^2\theta}=\frac{-5}{4x}\pi^3\tanh^{-1}x$

$$+\left(\frac{\pi-\pi^2}{x}\right)\tanh^{-1}x+\left(\frac{\pi}{2x}\beta(\frac{1}{2},\frac{2n+1}{2})((-1)^{2n}+1)\right)\tanh^{-1}x-\frac{\pi}{2x}\sum_{n=0}^{\infty}\left[\frac{x^{2n+1}}{(n+1)}\left(1+\frac{\beta(\frac{1}{2},\frac{2n+1}{2})((-1)^{2n}+1)}{2}\right)\right]$$

Proof: Consider $I_3=\int_0^{\pi}\frac{\theta^3\sin\theta d\theta}{1-x^2\cos^2\theta}$

$$=\int_0^{\pi}\frac{(\pi-\theta)^3\sin\theta d\theta}{1-x^2\cos^2\theta}$$

$$=\pi^3\int_0^{\pi}\frac{\sin\theta d\theta}{1-x^2\cos^2\theta}-3\pi^2\int_0^{\pi}\frac{\theta\sin\theta d\theta}{1-x^2\cos^2\theta}+3\pi\int_0^{\pi}\frac{\theta^2\sin\theta d\theta}{1-x^2\cos^2\theta}-\int_0^{\pi}\frac{\theta^3\sin\theta d\theta}{1-x^2\cos^2\theta}$$

By using Lemma 3, we get

$$=\pi^3\int_0^{\pi}\frac{\sin\theta d\theta}{1-x^2\cos^2\theta}-3\pi^2\int_0^{\pi}\frac{\theta\sin\theta d\theta}{1-x^2\cos^2x}-I_3$$

$$+3\left(\frac{\pi-\pi^2}{x}+\frac{\pi}{2x}\beta(\frac{1}{2},\frac{2n+1}{2})((-1)^{2n}+1)\right)\tanh^{-1}x-\frac{\pi}{2x}\sum_{n=0}^{\infty}\left[\frac{x^{2n+1}}{(n+1)}\left(1+\frac{\beta(\frac{1}{2},\frac{2n+1}{2})((-1)^{2n}+1)}{2}\right)\right]$$

$$=>I_3=\frac{1}{2}\left[\pi^3\int_0^{\pi}\frac{\sin\theta d\theta}{1-x^2\cos^2\theta}-3\pi^2\int_0^{\pi}\frac{\theta\sin\theta d\theta}{1-x^2\cos^2x}\right]$$

$$=\frac{-5}{4x}\pi^3\tanh^{-1}x$$

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$+\left(\frac{\pi-\pi^2}{x}+\frac{\pi}{2x}\beta\left(\frac{1}{2},\frac{2n+1}{2}\right)\left((-1)^{2n}+1\right)\right)$
$tanh^{-1}x$

$-\frac{\pi}{2x}\sum_{n=0}^{\infty}\left[\frac{x^{2n+1}}{(n+1)}\left(1+\frac{\beta\left(\frac{1}{2},\frac{2n+1}{2}\right)\left((-1)^{2n}+1\right)}{2}\right)\right]$ …(1.4)

Lemma 4: A similar generalization is also given in [4], which is as follows

$\int_0^{\pi}\frac{\theta(sin\theta)^3 d\theta}{1-x^2cos^2\theta}=\sum_{n=0}^{\infty}\frac{x^{2n}\pi}{(2n+1)(2n+3)}$

$=\frac{\pi}{x^2}\left[1+\frac{x^2-1}{x}tanh^{-1}x\right]$

Lemma 5: [4] For integer m>0,

$$\int_0^{\pi}\theta sin^{2j+1}\theta cos^{2m}\theta d\theta=\frac{2^j j!\,\pi}{(2m+1)(2m+3)(2m+5)\ldots..(2m+2j+1)}$$

Using 7 and 8(Lemma 4 and Lemma 5), generalized R_2 solution is obtained in [4] as

$$\int_0^{\pi}\theta\frac{sin^{2j+1}\theta}{1-x^2cos^2\theta}d\theta$$

$$=\sum_{n=0}^{\infty}\frac{2^j j!\pi x^{2n}}{(2m+1)(2m+3)(2m+5).....(2m+2j+1)}$$

For further generalization we consider,

$$\int_0^{\pi}\theta\frac{sin\theta}{(1-x^2cos^2\theta)^m}d\theta$$

$$=\int_0^{\pi}\theta sin\theta\left[1+\sum_{n=0}^{\infty}\frac{\prod_{i=0}^{n-1}(m+i)\,x^{2n}(cos\theta)^{2n}}{(n-1)!}\right]d\theta$$

$$=\int_0^{\pi}\theta sin\theta d\theta+\sum_{n=0}^{\infty}\frac{\prod_{i=0}^{n-1}(m+i)x^{2n}}{(n-1)!}$$

$$\int_0^{\pi}\theta sin\theta\,(cos\theta)^{2n}d\theta$$

$$=\pi+\sum_{n=0}^{\infty}\frac{\prod_{i=0}^{n-1}(m+i)x^{2n}}{(n-1)!}\frac{\pi}{(2n+1)}$$

Further continuing we obtain

$$\int_0^{\pi}\theta\frac{sin^{2j+1}\theta}{(1-x^2cos^2\theta)^m}d\theta$$

$$=\int_0^{\pi}\theta(sin\theta)^{2j+1}\left[1+\sum_{n=0}^{\infty}\frac{\prod_{i=0}^{n-1}(m+i)\,x^{2n}(cos\theta)^{2n}}{(n-1)!}\right]d\theta$$

$$=\int_0^{\pi}\theta(sin\theta)^{2j+1}d\theta+\int_0^{\pi}\theta(sin\theta)^{2j+1}$$

$$\sum_{n=0}^{\infty}\frac{\prod_{i=0}^{n-1}(m+i)x^{2n}(cos\theta)^{2n}}{(n-1)!}d\theta$$

$$=\int_0^{\pi}\theta(sin\theta)^{2j+1}d\theta+\sum_{n=0}^{\infty}\frac{\prod_{i=0}^{n-1}(m+i)x^{2n}}{(n-1)!}$$

$$\int_0^{\pi}\theta(sin\theta)^{2j+1}(cos\theta)^{2n}d\theta$$

$$=\frac{\pi}{2}\beta(j+1,1/2)+\sum_{n=0}^{\infty}\frac{\prod_{i=0}^{n-1}(m+i)x^{2n}}{(n-1)!}$$

$$\beta\left(j+1,\ \frac{2n+1}{2}\right)$$

Further continuing

$$\int_0^{\pi}\theta\frac{sin^{2j+1}\theta}{(1-x^2cos^2\theta)^{m/2}}d\theta$$

$$=\int_0^{\pi}\theta(sin\theta)^{2j+1}\left[1+\sum_{n=0}^{\infty}\frac{\prod_{i=0}^{n-1}(m+2*i)\,x^{2n}(cos\theta)^{2n}}{(n)!*2^n}\right]d\theta$$

$$=\int_0^{\pi}\theta(sin\theta)^{2j+1}d\theta+\sum_{n=0}^{\infty}\frac{\prod_{i=0}^{n-1}(m+2*i)x^{2n}}{(n)!*2^n}$$

$$\int_0^{\pi}\theta(sin\theta)^{2j+1}(cos\theta)^{2n}d\theta$$

$$=\frac{\pi}{2}\beta(j+1,1/2)+\sum_{n=0}^{\infty}\frac{\prod_{i=0}^{n-1}(m+2*i)x^{2n}}{(n)!*2^n}\beta$$

Now another result we may consider,

$I(x)=\int_0^{\pi}\frac{\theta sin\theta d\theta}{1-x^2cos^2\theta}$ …(1.6)

Differentiate w.r.t x then

$$I'(x)=\int_0^{\pi}\frac{\theta sin\theta(2xcos^2\theta)d\theta}{(1-x^2cos^2\theta)^2}$$

$$=\frac{-2}{x}\int_0^{\pi}\frac{(\theta sin\theta(1-x^2cos^2\theta)-\theta sin\theta)d\theta}{(1-x^2cos^2\theta)^2}$$

$$=\frac{2}{x}\int_0^{\pi}\frac{\theta sin\theta d\theta}{(1-x^2cos^2\theta)^2}-\frac{2}{x}I(x)$$

Hence, $\frac{dI(x)}{dx}+\frac{2}{x}I(x)=\frac{-2}{x}\int_0^{\pi}\frac{\theta sin\theta d\theta}{(1-x^2cos^2\theta)^2}$

$$=\frac{-2}{x}\int_0^{\pi}\theta sin\theta\sum_{n=0}^{\infty}(n+1)(cos\theta)^{2n}(x)^{2n}d\theta$$

So $\frac{dI(x)}{dx}+\frac{2}{x}I(x)$

$$=\frac{-2}{x^2}\sum_{n=0}^{\infty}(n+1)(x)^{2n+1}$$

$$\int_0^{\pi}\theta sin\theta(cos\theta)^{2n}d\theta$$

$$=\frac{-2}{x^2}\sum_{n=0}^{\infty}(n+1)(x)^{2n+1}\frac{\pi}{2n+1}$$

$$=\frac{-2\pi}{x^2}\sum_{n=0}^{\infty}\frac{(n+1)(x)^{2n+1}}{2n+1}$$

$$=-\frac{-2\pi}{x^2}\sum_{n=0}^{\infty}(x)^{2n+1}\frac{(n+1)}{2n+1}$$

$$=\frac{-2\pi}{x^2}\sum_{n=0}^{\infty}(x)^{2n+1}\left[1+\frac{1}{2n+1}\right]$$

$$=\frac{-2\pi}{x^2}\sum_{n=0}^{\infty}\left[(x)^{2n+1}+\frac{(x)^{2n+1}}{2n+1}\right]$$

$$=\frac{-2\pi}{x^2}\left[\sum_{n=0}^{\infty}(x)^{2n+1}+\sum_{n=0}^{\infty}\frac{(x)^{2n+1}}{2n+1}\right]$$

$$=\left[\frac{-2\pi}{x}\sum_{n=0}^{\infty}(x)^{2n}+\frac{-2\pi}{x^2}\sum_{n=0}^{\infty}\frac{(x)^{2n+1}}{2n+1}\right]$$

$=\frac{-2\pi}{x(1-x^2)}-\frac{2\pi}{x^2}tanh^{-1}x$ by lemma1.

Hence,

$\frac{dI(x)}{dx}+\frac{2}{x}I(x)=\frac{-2\pi}{x(1-x^2)}-\frac{2\pi}{x^2}tanh^{-1}(x)$ …(1.7)

Solving the above First order ordinary linear differential equation (1.7) using, Maxima software, we get the integrating factor as x^2, and the solution is

$$x^2I(x)=-2\pi\int x^2\left(\frac{1}{x(1-x^2)}-\frac{tanh^{-1}x}{x^2}\right)dx$$

$$I(x)=\frac{2\pi}{x^2}\left(log(1-x^2)-xtanh^{-1}x\right)$$

II. CONCLUSION

The above results of integration are still in the initial stage, but give rise to several types' integral forms, and needs more focus of verification. It also opens multiple, distinct types of food for researchers.

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Anand G Puranik, received his M Sc, M Phil and PhD degrees from Karnatak University Dharwad and he is presently working as Assistant Professor in Government Science College Chitradurga, His earlier research works were in Entire and Meromorphic Functions concerning exceptional values of Differential Equations. Anand G Puranik, taught many branches of Mathematics like Differential Equations, Complex Analysis, Topology, Vector Calculus, Differential and Integral Calculus etc for more than two decades. He also earned Post Graduate Diploma in Computer Application, and also taught C programming, Elementary Statisstics like Regression and Correlations, Probability, Probability Distributions and Testing of Hypothesis. This is his first research article out of his earlier interest. He strongly believes that the new view introduced in this article gives a booming effect for the further research in Integral Calculus.

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