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Algebraizations of Propositional Logic and Monadic Logic

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***Abstract:** In this paper, we introduce the language systems of propositional logic (LSPL), which involves no variables, and monadic predicate logic (LSMPL), which consists of predicates applied to single variables. We review the validity and dedication related to (LSPL) and (LSMPL) with their properties. After that, we investigate the connection between Boolean algebras with (LSPL) and (LSMPL) to make algebraizations methods out of logic.*

***Keywords**: Propositional and Monadic Logic, Validity, Deduction, Boolean Algebra, Algebraizations of Propositional and Monadic Logic.*

I. INTRODUCTION

The current paper sheds light on the link between propositional logic (LSPL) and monadic predicate logic (LSMPL) with Boolean algebras. The father of algebraic logic is George Bool who introduced Boolean algebras in the 1850's to express statement logic in algebraic form [2]. In [1], we studied propositional logic with important characteristics. In [2] we investigated and enlarged existential and universal quantifier operators on Boolean algebras with their properties. In [3], the extension of monadic and their properties by introducing ideals, filters, homomorphism, and constant mapping and derived some results which associate ideal filters under mapping homomorphism.

II. PROPOSITIONAL LOGIC

A. Validity of language for propositional logic system (VLPLS)

In this section, we review the basic concepts of propositional logic, for more information see [1,5,4,8,18,19,20,21,22,23,24,25,26,27].

Definition (1). The language L of system propositional logic (LSPL) consists of:

1. Symbols $p_1, p_2, p_3, \dots$ (for simple proposition);
2. Symbols $\neg, \wedge, \vee, \longrightarrow, \longleftrightarrow$ (for connective proposition) and
3. Punctuation $($, $)$.

Definition (2). A well- formed formula (**wff**) of language LSPL is defined as follows:

i. $P_1, P_2, P_3, \dots$ are well- formed formulas.

ii. If A and B are **wffs,** then $\neg A$, $A \wedge B$, $A \vee B, A \longrightarrow B, A \longleftrightarrow B$ are **wffs**.

Remark. The Symbols $\neg, \wedge, \vee, \longrightarrow, \longleftrightarrow$ indicted to negation, conjunction, disjunction, conditional and biconditional respectively.

Definition (3). An argument form is a finite sequence of **wffs** $A_1, A_2, A_3, \dots, A_n$ is called premises followed by a **wff** B called conclusion. This is written as follows:

$A_1, A_2, A_3, \dots, A_n, \therefore B$

The main central problem in LSPL is how to check whether or not the conclusion B is derived from the given premises' argument form $A_1, A_2, A_3, \dots, A_n$? Usually, there are two different ways called validity and deduction to do this.

The truth value of any **wff** in LSPL is considered true " T" or false "F" but not both. This is the principle of bivalence of classical logic.

Definition (4). A valuation (truth assignment or interpretation) v in the language LSPL is a mapping from the set of simple proposition letters into the set $\{T, F\}$.that is,

$$v(p) = \begin{cases} T, if\ p\ is\ true \\ F, if\ p\ is\ false \end{cases}$$

which satisfies the following conditions:

1. $v(\neg p) \neq v(p)$;
2. $v(p \wedge q) = T \leftrightarrow v(p) = v(q) = T$;
3. $v(p \vee q) = F \leftrightarrow v(p) = v(q) = F$;
4. $v(p \rightarrow q) = F \leftrightarrow v(p) = T \wedge v(q) = F$, and
5. $v(p \leftrightarrow q) = T \leftrightarrow v(p) = v(q)$.

All interpretations of a **wff** can be viewed by a truth table.

Definition (5). An argument form $A_1, A_2, A_3, \dots, A_n, \therefore B$ is called valid if there is no interpretation v such that: $v(A_1) = v(A_2) = v(A_3) = \cdots v(A_n) = T\ and\ v(B) = F$ .The valid argument form is denoted by: $A_1, A_2, A_3, \dots, A_n \vDash B$, otherwise it is called invalid and denoted by:

$A_1, A_2, A_3, \dots, A_n \nvDash B$.

Theorem (1). $A_1, A_2, A_3, \dots, A_n \vDash B$ if and only if $(A_1 \wedge A_2 \wedge A_3 \wedge \dots \wedge A_n) \longrightarrow B$ is true for all interpretation v.

Definition (6). A **wff** B is called

1. Valid if $\vDash B$ i.e., $v(B) = T$ for any interpretation v.
2. Satisfiability (contingent) if $v(B) = T$ for some interpretation v.
3. Un Satisfiability contradiction) if $v(B) = F$ for any interpretation v.

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Theorem (2)

1. $A_1, A_2, A_3, \dots, A_n \vDash A \longrightarrow B$ if and only if $A_1, A_2, A_3, \dots, A_n, A \vDash B$.
2. $A_1, A_2, A_3, \dots, A_n \vDash B$ if and only if $A_1, A_2, A_3, \dots, A_n, \neg B \vDash 0$.
3. If $\vDash A$ and $\vDash A \longrightarrow B$, then $\vDash B$.
4. $\vDash A$ if and only if , $\neg A$ is a contradiction.

Definition (7). A **wff** A is a logically implies a **wff** B, if $A \vDash B$. This is denoted by $A \Longrightarrow B$ in language LSPL. Two **wffs** A and B are logically equivalents in language LSPL, if $A \Longrightarrow B$ and $B \Longrightarrow A$ which written as $B \Leftrightarrow A$. The relation $\Leftrightarrow$ is an equivalence relation on **wffs** of the language LSPL. The relation $\Longrightarrow$ is ordering relation on **wffs** up to logical equivalence.

B. Deduction of the Language for Propositional Logic System (DLPLS)

Definition (1). A rule of inference is a mapping that maps asset (possibly empty) of **wff** $\alpha_1, \alpha_2, \dots, \alpha_n$ into a **wff** β. It is written as follows: $\alpha_1, \alpha_2, \dots, \alpha_n \therefore / \beta$.

Definition (2). Let $A_1, A_2, A_3, \dots, A_n ; \therefore B$ be an argument form. We say that the conclusion B is deducible from the premises $A_1, A_2, A_3, \dots, A_n$. If there is a finite sequence of **wff**s such that:

i. Each **wff** of the sequences either belongs to $\{A_1, A_2, A_3, \dots, A_n\}$ or is derived from pervious **wff** in the sequence by an inference rule.
ii. The last **wff** of the sequence is B. The finite sequence of **wffs** is called natural deduction (proof) in LSPL, this is denoted by: $A_1, A_2, A_3, \dots, A_n \vdash B$ (this is also called sequent). B is called theorem in LSPL, if $\vdash B$.

Definition (3). A and B are called provably equivalent if $A \vdash B$ and $B \vdash A$,this is denoted by $A \dashv\vdash B$.

Theorem (1). [1] DE Morgan's theorems are provably equivalent. I.e., $\neg(A \wedge B) \dashv\vdash \neg A \vee \neg B$.

Theorem (2). Prove the argument $A \to B, \neg B \vdash \neg A$.

Proof.

Line #	wff	Reason
1.	$A \to B$	Pre
2.	$\neg B$	Pre
3.	A	Ass
4.	B	$1,3, \to -E$
5.	0	$2,4, \neg - E$
6.	$\neg A$	$3,5, \neg - I$ Discharge 3■.

Note that. The logicians known as the previous theorem by the rule of Modus Tollens.

Theorem (3). The following argument is valid.

$\vdash \neg\{[\neg(A \wedge B) \wedge \neg(A \wedge \neg B)] \wedge [\neg(\neg A \wedge B) \wedge \neg(\neg A \wedge \neg B)]\}$.

Proof.

Line #	wff	Reason
1.	$\left\{\begin{matrix}[\neg(A \wedge B) \wedge \neg(A \wedge \neg B)] \wedge \\ [\neg(\neg A \wedge B) \wedge \neg(\neg A \wedge \neg B)]\end{matrix}\right\}$	Ass
2.	$[\neg(A \wedge B) \wedge \neg(A \wedge \neg B)]$	$1, \wedge - E$
3.	$[\neg(\neg A \wedge B) \wedge \neg(\neg A \wedge \neg B)]$	$1, \wedge - E$
4.	$\neg(A \wedge B)$	$2, \wedge - E$
5.	$\neg(A \wedge \neg B)$	$2, \wedge - E$
6.	$\neg(\neg A \wedge B)$	$3, \wedge - E$
7.	$\neg(\neg A \wedge \neg B)$	$3, \wedge - E$
8.	$A \to \neg B$	4, by thm
9.	$A \to \neg\neg B$	5, by thm
10.	A	Ass
11.	$\neg B$	$8,10, \to -E$
12.	$\neg\neg B$	$9,10, \to -E$
13.	0	$11,12, \neg - E$
14.	$\neg A$	$10,13, \neg - I$
15.	$\neg A \to \neg B$	6, by thm
16.	$\neg A \to \neg\neg B$	7, by thm
17.	$\neg A$	Ass
18.	$\neg B$	$15,17, \to -E$
19.	$\neg\neg B$	$16,17, \to -E$
20.	0	$18,19, \neg - E$
21.	$\neg\neg A$	$17,20, \neg - I$
22.	0	$14,21, \neg - I$
23.	$\neg\left\{\begin{matrix}[\neg(A \wedge B) \wedge \neg(A \wedge \neg B)] \wedge \\ [\neg(\neg A \wedge B) \wedge \neg(\neg A \wedge \neg B)]\end{matrix}\right\}$	$1,22, \neg - I$■.

III. ALGEBRAIZATIONS OF PROPOSITIONAL LOGIC

In this section, we present the main result how to make algebra out of logic due to Halmos, see [2,10,11,12,13,15,16]. Moreover, to know about the facts of Boolean algebra, see [2,6,9,14,17].

Definition (1). [2,6,14] A Boolean Algebra is an algebraic structure $\mathcal{B} = (B, \vee, \wedge, ', 0, 1)$ consists of a set B, two binary operations $\vee$ (join) and $\wedge$ (meet), one unary operation '(complementation) and two nullary operations 0 and 1(fixed element) which satisfies the following axioms:

BA_1. $a \vee b = b \vee a$ and $a \wedge b = b \wedge a, \forall\, a, b \in B$

(commutative axiom);

BA_2. $(a \vee b) \vee c = a \vee (b \vee c)$ and

$(a \wedge b) \wedge c = a \wedge (b \wedge c), \forall\, a, b, c \in B$

(associative axiom);

BA_3. $a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$ and

$a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c),\ \forall\, a, b, c \in B$

(distributive axioms);

BA_4. $a \vee a = a$ and $a \wedge a = a$ (idempotent axiom);

BA_5 . $a \vee (a \wedge b) = a$ and $a \wedge (a \vee b) = a, \forall\, a, b \in B$

(absorption axiom);

BA_6. $a \wedge 1 = a$ and $a \vee 0 = a, \forall\, a \in B$

(existence of zero and unit elements axioms), and

BA_7. $\forall\, a \in B \Longrightarrow \exists\, a' \in B \ni a \vee a' = 1$ and

$a \wedge a' = 0$ (existence of complment axiom).

The following theorems gives us the main properties of elements of Boolean Algebras.

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Now, Consider a set $S = \{p : p \text{ is a } \mathbf{wff}\}$ of all **wffs**. Define the relation $\Leftrightarrow$ on S as follows: $p \Leftrightarrow q$ if and only if $\vDash (p \leftrightarrow q)$ It easy to verify that $\Leftrightarrow$ is an equivalence relation on S. Let $B = S/_{\Leftrightarrow} = \{[p] : p \in S\}$ be the quotient set. Define a binary relation $\leq$ on B as follows: $[p] \leq [q]$ if and only if $p \Longrightarrow q$ (**iff** $\vDash p \to q$). $\leq$ is well- defined and ordering relation on B. Define operations on B as follows:

i. $[p] \vee [q] = [p \vee q]$.

ii. $[p] \wedge [q] = [p \wedge q]$.

$[p'] = [\neg p]$, for all $[p], [q] \in B$.

Moreover, let $[0] = [p \wedge \neg p]$ and $[1] = [p \vee \neg p]$, these are well-defined.

Theorem (1). The Algebraic structure $\mathcal{B} = (B, \vee, \wedge, ', [0], [1])$ is a Boolean Algebra,

Proof. Let $[p], [q]$ and $[r] \in B$, then:

BA_1. $[p] \vee [q] = [p \vee q] = [q \vee p] = [q] \vee [p]$ and $[p \wedge q] = [p \wedge q] = [q \wedge p] = [q] \wedge [p], \forall [p], [q] \in B$ (commutative axiom).

BA_2. $([p] \vee [q]) \vee [r] = [p \vee q] \vee [r]$

$$= [(p \vee q) \vee r]$$
$$= [p \vee (q \vee r)]$$
$$= [p] \vee [q \vee r] = [p] \vee ([q] \vee [r]),$$

and $([p] \wedge [q]) \wedge [r] = [p \wedge q] \wedge [r]$

$$= [(p \wedge q) \wedge r]$$
$$= [p \wedge (q \wedge r)]$$
$$= [p] \wedge [q \vee r] = [p] \wedge ([q] \wedge [r]),$$

$\forall [p], [q], [r] \in B$ ((associative axiom).

BA_3. $[p] \vee ([q] \wedge [r]) = [p] \vee [q \wedge r]$

$$= [p \vee (q \wedge r)]$$
$$= [(p \vee q) \wedge (p \vee r)]$$
$$= [p \vee q] \wedge [p \vee r]$$
$$= ([p] \vee [q]) \wedge ([p] \vee [r]) \text{ and}$$

$[p] \wedge ([q] \vee [r]) = ([p] \wedge [q]) \vee ([p] \wedge [r])$,

$\forall [p], [q], [r] \in B$ (distributive axiom). By similar method.

BA_4. $[p] \vee [p] = [p \vee p] = [p]$ and

$[p] \wedge [p] = [p \wedge p] = [p]$ (idempotent axiom).

BA_5. $[p] \vee ([p] \wedge [q]) = [p] \vee [p \wedge q]$

$$= [p \vee (p \wedge q)] = [p], \text{ and}$$

$[p] \wedge ([p] \vee [q]) = [p] \wedge [p \vee q]$

$$= [p \wedge (p \vee q)] = [p],$$

$\forall [p], [q] \in B$ (absorption axiom).

BA_6. $[p] \wedge [1] = [p \wedge 1] = [p]$. and

$[p] \vee [0] = [p \vee 0] = [p], \forall [p] \in B$.

(existence of zero and unit elements axioms) and

(existence of zero and unit elements axioms) and

BA_7. $\forall [p] \in B \Longrightarrow \exists [p]' \in B$ such that

$[p] \vee [p]' = [p] \vee [\neg p] = [p \vee \neg p] = [1]$ and

$[p] \wedge [p]' = [p] \wedge [\neg p] = [p \wedge \neg p] = [0]$

(existence of complment axiom) ■.

A propositional logic can now be represented by Boolean algebras, where the equivalence classes of **wffs** of a statement are represented by elements of the Boolean algebra. The logical operations $\neg, \wedge, \vee, \longrightarrow, \leftrightarrow$ have their Boolean counterparts. The Logical relations $\Leftrightarrow$ and $\Longrightarrow$ are represented by $=$ and $\leq$, respectively, thus sequent in propositional logic can be algebraic proofs.

Remark. In logic $\Leftrightarrow$ corresponding to $\dashv\vdash$ and $\Longrightarrow$ corresponding to $\vdash$ if we pass from validity to deduction.

Theorem (2). The **wff,** $\neg(A \wedge B) \dashv\vdash \neg A \vee \neg B$ in $LSPL$ see [1] corresponding to $(a \wedge b)' = (a' \vee b')$ in Boolean algebra B. We can prove algebraically that $(a \wedge b)' = (a' \vee b')$.

Proof.

$$(a \wedge b) \vee (a' \vee b') = \{a \vee (a' \vee b')\} \wedge \{b \vee (a' \vee b')\}$$
$$= \{(a \vee a') \vee b'\} \wedge \{b \vee (b' \vee a')\}$$
$$= \{(a \vee a') \vee b'\} \wedge \{(b \vee b') \vee a'\}$$
$$= \{1 \vee b'\} \wedge \{1 \vee a'\}$$
$$= 1 \wedge 1 = 1.$$

From another hand, we have,

$$(a \wedge b) \wedge (a' \vee b') = \{(a \wedge b) \wedge a'\} \vee \{(a \wedge b) \wedge b'\}$$
$$= \{(b \wedge a) \wedge a'\} \vee \{(a \wedge b) \wedge b'\}$$
$$= \{b \wedge (a \wedge a')\} \vee \{a \wedge (b \wedge b')\}$$
$$= \{b \wedge (a \wedge a')\} \vee \{a \wedge (b \wedge b')\}$$
$$= \{b \wedge 0\} \vee \{a \wedge 0\}$$
$$= 0 \vee 0 = 0.$$

Therefore $(a \wedge b)' = (a' \vee b')$ ■.

Theorem (3). The **wff**, $A \to B, \neg B \vdash \neg A$ in $LSPL$ corresponding to $(a' \vee b) \wedge b' \leq a'$ in Boolean algebra.

Proof.

$$a \to b \wedge b' = (a' \vee b) \wedge b' = (a' \wedge b') \vee (b \wedge b') = (a' \wedge b') \vee 0 = a' \wedge b' \leq a' \ ■.$$

Theorem (4). $p \longrightarrow q \Leftrightarrow \neg q \longrightarrow \neg p$ in $LSPL$. This corresponding to in Boolean algebra B,

by $a \longrightarrow b = b' \longrightarrow a'$,

Proof. since, $a \longrightarrow b = a' \vee b = b \vee a' = (b')' \vee a' = b' \longrightarrow a'$ ■.

Theorem (5). The **wff,** $(A \longrightarrow B), A \vdash B$ in $LSPL$ known as in dedication system of propositional logic (DSPL) by rule if…, then …, elimination $(\to -E)$,also, it is called Modus ponens, this is becoming in a Boolean algebra $(a' \vee b) \wedge a) \leq b$.

Proof.

$(a \longrightarrow b) \wedge a = (a' \vee b) \wedge a = (a' \wedge a) \vee (b \wedge a) = 0 \vee (b \wedge a) = (b \wedge a) \leq b$ ■.

Theorem (6). The argument:

$\vdash \neg\{[\neg(A \wedge B) \wedge \neg(A \wedge \neg B)] \wedge [\neg(\neg A \wedge B) \wedge \neg(\neg A \wedge \neg B)]\}$

is corresponding to

$\{[(a \wedge b)' \wedge (a \wedge b')'] \wedge [(a' \wedge b)' \wedge (a' \wedge b')']\}' = 1.$

in Boolean algebra:

Proof.

$\{[(a \wedge b)' \wedge (a \wedge b')'] \wedge [(a' \wedge b)' \wedge (a' \wedge b')']\}'$

$= [(a \wedge b)' \wedge (a \wedge b')']' \vee [(a' \wedge b)' \wedge (a' \wedge b')']'$

$= [(a \wedge b) \vee (a \wedge b')] \vee [(a' \wedge b) \vee (a' \wedge b')]$

$= [a \wedge (b \vee b')] \vee [a' \wedge (b \vee b')]$

$= [a \wedge 1] \vee [a' \wedge 1]$

$= a \vee a' = 1$■.

IV. SAYTEM OF MONADIC PREDICATE LOGIC

Definition (1). A first-order language for system monadic predicate logic (LSMPL) consists of symbols for:

1. Constant letters $a_1, a_2, a_3, \dots$
2. Variables letters $x_1, x_2, x_3, \dots$
3. Functions letters $f_1, f_2, f_3, \dots$
4. Predicate letters $A_1, A_2, A_3, \dots$
5. Quantifiers $\forall, \exists$.
6. Connective symbols $\neg, \wedge, \vee, \longrightarrow, \longleftrightarrow$ and
7. Punctuation (,).

Definition (2). A term of first-order language for system monadic predicate logic (LSMPL) is defined as follows:

1. $a_1, a_2, a_3, \dots$ are terms,
2. $x_1, x_2, x_3, \dots$ are terms and
3. If t is a term, then $f_i(t)$ is a term.

Definition (3). An atomic formula of first order (LSMPL) defined as follows, if t is a term, then $A_i(t)$ is called an atomic formula.

Definition (4). A well- formed formula (**wff**) of first order (LSMPL) is defined as follows:

1. Any atomic formula is a **wff**, and
2. If A and B are **Wffs**, then $\neg A$, $A \wedge B$, $A \vee B$, $A \longrightarrow B$, $A \longleftrightarrow B$, $(\forall x_i)A$, and $(\exists x_i)A$ are **wffs**.

Definition (5). A variable x_i occurring in a **wff** is called a bound, if it is within the scope of quantifiers $(\forall x_i)$ or $(\exists x_i)$. Otherwise x_i is called free variable. A **wff** is called closed, if it has no free variables.

Definition (6). A term of first-order language for system monadic predicate logic (LSMPL) is defined as follows:

1. $a_1, a_2, a_3, \dots$ are terms,
2. $x_1, x_2, x_3, \dots$ are terms and
3. If t is a term, then $f_i(t)$ is a term.

Definition (7). An atomic formula of first order (LSMPL) defined as follows, if t is a term, then $A_i(t)$ is called an atomic formula.

Definition (8). A well- formed formula (**wff**) of first order (LSMPL) is defined as follows:

1. Any atomic formula is a **wff**, and
2. If A and B are **wffs**, then $\neg A$, $A \wedge B$, $A \vee B$, $A \longrightarrow B$, $A \longleftrightarrow B$, $(\forall x_i)A$, and $(\exists x_i)A$ are **wffs**.

Definition (9). A variable x_i occurring in a **wff** is called a bound, if it is within the scope of quantifiers $(\forall x_i)$ or $(\exists x_i)$. Otherwise x_i is called free variable. A **wff** is called closed, if it has no free variables.

Definition (10). If A is a formula and A accurse of formula of B then A is called a sub-formula of B.

Remark. Argument forms of first-order (LMPLS) are defined as in (LPLS). The (LMPLS) is more expressive than the (LSPL).

A. Validity of Monadic Predicate Logic System (VMPLS)

Definition (1). An interpretation I of (LSMPL) consist of a domain D_I of values such that:

1. $a_1, a_2, a_3, \dots$ correspond to a fixed value $\overline{a_1}, \overline{a_2}, \overline{a_3}, \dots$ in D_I,
2. Function symbols f_i correspond to any unary operation $\overline{f_i}: D_I \longrightarrow D_I$, and
3. Predicate symbols A_i correspond to any unary relation $\overline{A_i} \subseteq D_I$. Let I be an interpretation with a **wff** A of (LSMPL) then:

i. If A is closed **wff**, then its translation in I is a proposition (i.e. it is true or false).

ii. If A is constant free variable s, then its translation in I is an open sentence.

iii. The assignments of values from D_I to the free variables will make the open sentence satisfiability or not in I. If the open sentence is satisfiability for all assignments of values of D_I it is said to be true in I. A is called false in I, if it is not satisfiability by any an assignment of value of I.

iv. If A is true in every interpretation, then A is called valid and it is denoted by $\vDash A$. The validity of an argument $A_1, A_2, A_3, \dots, A_n ; \therefore B$ in first order (LSMPL) is denoted by $A_1, A_2, A_3, \dots, A_n \vDash B$ and is defined as in (LSPL) also the logical implication $\Longrightarrow$ and logical equivalence $\Longleftrightarrow$ are defined as mentioned in [7].

Remark. In classical logic the analyzed proposition into the following components

i. Subject term,

ii. Predicate term.

Theorem (1). Prove that the following argument in (LSMPL): $(\forall x)A(x) \vee (\forall x)B(x) \vDash (\forall x)\big(A(x) \vee B(x)\big)$ is valid, but the converse is invalid. That is,

$(\forall x)\big(A(x) \vee B(x)\big) \nvDash (\forall x)A(x) \vee (\forall x)B(x).$

Proof. Suppose that $D = \{a, b\}$. Now, $(\forall x)A(x) \equiv A(a) \wedge A(b) \equiv p \wedge q$, and $(\forall x)B(x) \equiv B(a) \wedge B(b) \equiv r \wedge s$. Hence the premises become: $p \wedge q) \vee (r \wedge s)$.

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Also the conclusion:

$$(\forall x)(A(x) \vee B(x)) \equiv (A(a) \vee B(a)) \wedge (A(b) \vee B(b)) \equiv (p \vee r) \wedge (q \vee s).$$

Now, we test the **wff**: $p \wedge q) \vee (r \wedge s) \rightarrow (p \vee r) \wedge (q \vee s)$. It is easy to verified that by truth table the **wff**: $(p \wedge q) \vee (r \wedge s) \rightarrow (p \vee r) \wedge (q \vee s)$ is a tautology. Therefore, the argument:

$$(\forall x)A(x) \vee (\forall x)B(x) \vDash (\forall x)(A(x) \vee B(x))$$ ■.

The converse of theorem by the following example.

Example (1). Show that the following argument: $(\forall x)(A(x) \vee B(x)) \nvDash (\forall x)A(x) \vee (\forall x)B(x)$ is invalid in (LSMPL), We want to show that: $(\forall x)(A(x) \vee B(x)) \rightarrow (\forall x)A(x) \vee (\forall x)B(x)$ invalid. Suppose that $D = \{a, b\}$ with the following interpretation as shown in Table.1.

Table of Disjunction Argument 1.

	A	B	$A \vee B$
a	T	F	T
b	F	T	T

The premise becomes:

$$(\forall x)(A(x) \vee B(x)) \equiv (A(a) \vee B(a)) \wedge (A(b) \vee B(b)) \equiv (T \vee F) \wedge (F \vee T) \equiv T \wedge T \equiv T.$$

Form anther hand, $(\forall x)A(x) \equiv A(a) \wedge A(b) \equiv T \wedge F \equiv F$ and $(\forall x)B(x) \equiv B(a) \wedge B(b) \equiv T \wedge F \equiv F$,hence the conclusion becomes, $(\forall x)A(x) \vee (\forall x)B(x) \equiv F \vee F \equiv F$. We see that: $T \rightarrow F \equiv F$. Hence,

$$(\forall x)(A(x) \vee B(x)) \nvDash (\forall x)A(x) \vee (\forall x)B(x).$$

Theorem (1). Prove that the following argument: $(\exists x)(A(x) \wedge B(x)) \vDash (\exists x)A(x) \wedge (\exists x)B(x)$ is valid in (LMPLS), but the converse is invalid, that is, $(\exists x)A(x) \wedge (\exists x)B(x) \nvDash (\exists x)(A(x) \wedge B(x))$

Proof. Suppose that $D = \{a, b\}$. We will prove $(\exists x)(A(x) \wedge B(x)) \rightarrow (\exists x)A(x) \wedge (\exists x)B(x)$ is valid argument. The premise represents by:

$$(\exists x)(A(x) \wedge B(x)) \equiv (A(a) \wedge B(a)) \vee (A(b) \wedge B(b)) \equiv (p \wedge r) \vee (q \wedge s)$$

and the conclusion represents by:

$(\exists x)A(x) \equiv A(a) \vee A(b) \equiv p \vee q$ and $(\exists x)B(x) \equiv B(a) \vee B(b) \equiv r \vee s$. Hence the conclusion becomes: $p \vee q) \wedge (r \vee s)$. So it easy to check **wff**: $(p \wedge r) \vee (q \wedge s) \rightarrow (p \vee q) \wedge (r \vee s)$ is valid by truth table and consequently,

$$(\exists x)(A(x) \wedge B(x)) \vDash (\exists x)A(x) \wedge (\exists x)B(x)$$ ■.

The following examples illustrates the converse of the theorem is invalid.

Example (2). Show that the following argument: $(\exists x)A(x) \vee (\exists x)B(x) \nvDash (\exists x)(A(x) \vee B(x))$ is invalid in (LMPLS. We need to show that: $(\exists x)A(x) \wedge (\exists x)B(x) \rightarrow (\exists x)(A(x) \wedge B(x))$ is an invalid. Suppose that $D = \{a, b\}$ with the following interpretation as shown in Table 2.

Table Conjunction Argument 2.

	A	B	$A \wedge B$
a	T	F	F
b	F	T	F

The premise becomes:

$(\exists x)A(x) \equiv A(a) \vee A(b) \equiv T \vee F \equiv T$ and $(\exists x)B(x) \equiv B(a) \vee B(b) \equiv T \vee F \equiv T$, hence $(\exists x)A(x) \wedge (\exists x)B(x) \equiv T \wedge T \equiv T$, the conclusion represents by:

$$(\exists x)(A(x) \vee B(x)) \equiv (A(a) \wedge B(a)) \vee (A(b) \wedge B(b)) \equiv (T \wedge F) \vee (F \wedge T) \equiv F \vee F \equiv F.$$

. we see that $T \rightarrow F \equiv F$,hence $(\exists x)A(x) \vee (\exists x)B(x) \nvDash (\exists x)(A(x) \vee B(x))$.

B. Dedication of Monadic Predicate Logic System (DMPLS)

A **wff** A involving x as a free variable may be denoted by $A(x)$. The inference rule of (DMPLS) consists of the rules of (DPLS) together with the following four rules, the inference rule of (DMPLS) consists of rules of (DPLS) together with the following four rules, $\forall - Elimination(\forall - E)$, $\forall - Introduction(\forall - I)$, $\exists - Elimination(\exists - E)$, and, $\exists - Introducation(\exists - I)$. The following theorems give us the main basic features of monadic logic see defined as definition 4.2.1 in [8], in addition,

Theorem (1). Inference rules of (LMPLS) are both sound and complete, that is: A **wff** A involving x as a free variable may be denoted by $A(x)$. The inference rule of (DMPLS) consists of the rules of (DPLS) together with the following four rules, the inference rule of (DMPLS) consists of rules of (DPLS) together with the following four rules, $\forall - Elimination(\forall - E)$, $\forall - Introduction(\forall - I)$, $\exists - Elimination(\exists - E)$, and, $\exists - Introducation(\exists - I)$. The following theorems give us the main basic features of monadic logic see defined as definition 4.2.1 in [8], in addition,

Theorem (2). Inference rules of (LMPLS) are both sound and complete, that is;

$A_1, A_2, A_3, \dots, A_n \vDash B$ **iff** $A_1, A_2, A_3, \dots, A_n \vdash B$.

Corollary (3). $\vDash B$ **iff** $A_1, A_2, A_3, \dots, A_n \vdash B$.

Theorem (4). (LMPLS) is consistent, incomplete and decidable.

Theorem (5). Prove that the following argument in (LMPLS) by (DMPLS): $(\forall x)A(x) \vee (\forall x)B(x) \vdash (\forall x)(A(x) \vee B(x))$

Proof.

Line #	wff	Reason
1.	$(\forall x)A(x) \vee (\forall x)B(x)$	Pre
2.	$(\forall x)A(x)$	Ass
3.	$A(t)$	$2, \forall - E$
4.	$A(t) \vee B(t)$	$3, \forall - I$
5.	$(\forall x)(A(x) \vee B(x))$	$4, \forall - I$
6.	$(\forall x)B(x)$	Ass
7.	$B(t)$	$6, \forall - E.$
8.	$A(t) \vee B(t)$	$7, \forall - I$
9.	$(\forall x)(A(x) \vee B(x))$	$8, \forall - I$
10.	$(\forall x)(A(x) \vee B(x))$	1,2,5,6,9, −E, discharge 2&6■.

Theorem (6). Prove the following arguments in (LMPLS) by (DMPLS):
$(\exists x)(A(x) \wedge B(x)) \vdash (\exists x)A(x) \wedge (\exists x)B(x)$.

Proof.

Line #	wff	Reason
1.	$(\exists x)(A(x) \wedge B(x))$	Pre
2.	$A(t) \wedge B(t)$	Ass
3.	$A(t)$	$2, \wedge - E$
4.	$B(t)$	$2, \wedge - E$
5.	$(\exists x)A(x)$	$3, \exists - I$
6.	$(\exists x)B(x)$	$4, \exists - I$
7.	$(\exists x)A(x) \wedge (\exists x)B(x)$	$5,6, \wedge - I$.
8.	$(\exists x)A(x) \wedge (\exists x)B(x)$	$1,2,7, \exists - E$, discharge 2 ■.

For the converse of pervious theorems (6) and (5). It is enough by Theorem (2). to show that:
$(\forall x)(A(x) \vee B(x)) \not\models (\forall x)A(x) \vee (\forall x)B(x)$, and
$(\exists x)A(x) \wedge (\exists x)B(x) \not\models (\exists x)(A(x) \wedge B(x))$, and consequently, the two examples (1) and (2) in IV-A. Moreover, we may consider the following example.

Example (1). Consider the following interpretation $D_1 = \mathbb{Z}^+ = \{0,1,2,\ldots\}$. Now, define $A(x) :=$ to be " x is even " and $B(x) :=$ to be " x is odd ". So,
$(\forall x)(A(x) \vee B(x)) \equiv (\forall x)(x \text{ is even } \vee x \text{ is odd}) \equiv T$
for all $D_1 = \mathbb{Z}^+$, from another hand,
$(\forall x)A(x) \equiv (x$ is even; $\forall x \in \mathbb{Z}^+) \equiv F$ and $(\forall x)B(x) \equiv (x$ is odd; $\forall x \in \mathbb{Z}^+) \equiv F$. Hence $(\forall x)A(x) \vee (\forall x)B(x) \equiv F \vee F \equiv F$, we deduced that, $T \rightarrow F \equiv F$. By Similar reasoning related to existential quantifier, we have $(\exists x)A(x) \wedge (\exists x)B(x) \equiv T$ and $(\exists x)(A(x) \wedge B(x)) \equiv F$, hence
$T \rightarrow F \equiv F$.

Theorem (7). Prove that the following argument in (LMPLS) by (DMPLS):
$(\forall x)(A(x) \rightarrow B(x)), (\forall x)A(x) \vdash (\forall x)B(x)$.

Proof.

Line #	wff	Reason
1.	$(\forall x)(A(x) \rightarrow B(x))$	Pre
2.	$(\forall x)A(x)$	Pre
3.	$A(t) \rightarrow B(t)$	$1, \forall - E$
4.	$A(t)$	$2, \forall - E$
5.	$B(t)$	$3,4, \rightarrow - E$
6.	$(\forall x)B(x)$	$5, \forall - I$ ■.

Theorem (8). Prove that the following in (LMPLS) by (DMPLS): $(\exists x)(A(x) \rightarrow B) \vdash (\forall x)A(x) \rightarrow B$.

Proof.

Line #	wff	Reason
1.	$(\exists x)(A(x) \rightarrow B)$	Pre
2.	$A(t) \rightarrow B$	Ass
3.	$(\forall x)A(x)$	Ass
4.	$A(t)$	$3, \forall - E$
5.	B	$2,4, \rightarrow - E$
6.	$(\forall x)A(x) \rightarrow B$	$3,5, \rightarrow - I$, discharge-3
7.	$(\forall x)A(x) \rightarrow B$	$1,2,6, \exists - E$, discharge-2■.

Theorem (9). Prove that the following argument in (LMPLS) by (DMPLS): $\vdash (\exists x)((\exists y)A(y) \rightarrow A(x))$.

Proof.

Line #	wff	Reason
1.	$\neg(\exists x)((\exists y)A(y) \rightarrow A(x))$.	Ass
2.	$(\forall x)\neg((\exists y)A(y) \rightarrow A(x))$.	1, thm-list-seq-6(DE Morgan)
3.	$\neg((\exists y)A(y) \rightarrow A(t))$	$2, \forall - E$
4.	$\neg(\neg(\exists y)A(y) \vee A(t))$	$3, thm$
5.	$(\exists y)A(y) \wedge \neg A(t)$	$4, thm$(DE Morgan)
6.	$\neg A(t)$	$5, \wedge - E$
7.	$(\exists y)A(y)$	$5, \wedge - E$
8.	$(\exists x)A(x)$	7, thm-list-seq-2
9.	$A(t)$	Ass
10.	0	$6,9, \neg - E$
11.	0	$8,9,10, \exists - E$
12.	$\neg\neg(\exists x)((\exists y)A(y) \rightarrow A(x))$.	$1,11, \neg - I$
13.	$(\exists x)((\exists y)A(y) \rightarrow A(x))$	$12, DN$■.

Theorem (10). Prove that the following argument in (LMPLS) by (DMPLS):
$(\forall x)(A \vee B(x)) \dashv\vdash A \vee (\forall x)B(x)$

Proof. Frist direction: $(\forall x)(A \vee B(x)) \vdash A \vee (\forall x)B(x)$

Line #	wff	Reason
1.	$(\forall x)(A \vee B(x))$	Pre
2.	$A \vee B(t)$	$1, \forall - E$
3.	$\neg(A \vee (\forall x)B(x))$	Ass.
4.	$\neg A \wedge \neg(\forall x)B(x)$	$3, thm$(Demorgan)
5.	$(\exists y)A(y) \wedge \neg A(t)$	$4, thm$(DE Morgan)
6.	$\neg A(t)$	$5, \wedge - E$
7.	$(\exists y)A(y)$	$5, \wedge - E$
8.	$(\exists x)A(x)$	7, thm-list-seq-2
9.	$A(t)$	Ass
10.	0	$6,9, \neg - E$
11.	0	$8,9,10, \exists - E$
12.	$\neg\neg(\exists x)((\exists y)A(y) \rightarrow A(x))$.	$1,11, \neg - I$
13.	$(\exists x)((\exists y)A(y) \rightarrow A(x))$	$12, DN$

The second direction by same argument, therefore,
$(\forall x)(A \vee B(x)) \dashv\vdash A \vee (\forall x)B(x)$■.

V. ALGEBRAIZATIONS OF MONADIC LOGIC

A Boolean Algebra is complete if for any subset of it, it has supremum and infimum in the Boolean algebra. Assume that B is a complete Boolean algebra. Let X be a non-empty set which represents the domain of a monadic logic.
$B^X = \{p | p: X \longrightarrow B \text{ is a function }\}$, the set of all functions from X into B, is a functional Boolean algebra. This is so by Defining Boolean operators pointwise, see [1,2]. Elements of B^X are the form $p(x)$, where $p: X \longrightarrow B$ is function. These elements represent 1-place predicates of the monadic logic.

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Now, the existential and universal functional quantifiers of B^X Can be defined algebraically as follows: $(\exists x)p(x) = \sup_{x\in X}\{p(x)\}$ and $(\forall x)p(x) = \inf_{x\in X}\{p(x)\}$, $(\exists x)p(x)$ and $(\forall x)p(x)$ are exists, since B is complete. There are two functional quantifiers which represents existential and universal quantifiers of monadic logic.

Not that.

$$\begin{aligned}((\exists x)p'(x))' &= \left(\sup_{x\in X}\{p'(x)\}\right)' \\ &= \left(\left(\inf_{x\in X}\{p(x)\}\right)'\right)' \\ &= \inf_{x\in X}\{p(x)\} = (\forall x)p(x).\end{aligned}$$

Theorem (1). An existential functional quantifier is an existential quantifier.

Proof.

1. $(\exists x)0(x) = \sup\{0(x): x \in X\} = \sup\{0\} = 0 = 0(x)$.
2. $(\exists x)p(x) = \sup\{p(x): x \in X\} \geq p(x)$.
3. $(\exists x)(p(x)) \wedge (\exists x)(q(x)) = \sup_{x\in X}\{p(x) \wedge (\exists x)q(x)\}$

$$= \sup_{x\in X}\{p(x)\} \wedge (\exists x)q(x)\}$$
$$=(\exists x)p(x) \wedge (\exists x)q(x) \ \blacksquare.$$

Corollary (1). M with an existential functional quantifier is monadic algebra. The following tables illustrates the corresponding monadic systems and the functional is a monadic algebra.

Monadic Logic	Monadic Algebra
Variables $x_1, x_2, x_3, \ldots$	Variables letters $x_1, x_2, x_3, \ldots$
Constants $a_1, a_2, a_3, \ldots$	Constants $a_1, a_2, a_3, \ldots$
1-place predicate $p(x)$	Single variable function $p(x)$ *from* $X \longrightarrow B$.
$\neg$Negation	' complementation
$\wedge$ and	$\wedge$ meet
$\vee$ or	$\vee$ joint
$\longrightarrow$ conditional	$\longrightarrow$
$\longleftrightarrow$ biconditional	$\longleftrightarrow$
$\Leftrightarrow$	= equality
$\Rightarrow$	$\leq$ precede or equal
$(\exists x)$	$\sup_x$
$(\forall x)$	$\inf_x$

It is possible to introduce now algebraic proofs of the sequent of monadic logic

1. $(\forall x)p \Leftrightarrow p$ in monadic logic this becomes in monadic algebra $(\forall x)p = \inf_x\{p\} = p$, since p is a constant functional element.
2. $(\exists x)p \Leftrightarrow p$ ' by similar argument".
3. $(\forall x)p(x) \Leftrightarrow (\forall y)p(y)$ in monadic logic corresponding to the monadic algebra

   $(\forall x)p(x) = \inf_x\{p(x)\} = \inf_y\{p(y)\} = (\forall y)p(y)$.

   This possible as for as x and y are independent free variables.
4. $(\exists x)p(x) \Leftrightarrow (\exists y)p(y)$ " by similar argument 3".
5. $(\forall x)(p(x) \wedge q(x)) \Leftrightarrow (\forall x)p(x) \wedge (\forall x)q(x)$.
6. $(\exists x)(p(x) \wedge q(x)) \Leftrightarrow (\exists x)p(x) \wedge (\exists x)q(x)$.
7. $(\forall x)(p(x) \vee q(x)) \Leftrightarrow (\forall x)p(x) \vee (\forall x)q(x)$.
8. $(\exists x)(p(x) \vee q(x)) \Leftrightarrow (\exists x)p(x) \vee (\exists x)q(x)$.
9. $(\forall x)(p \vee q(x)) \Leftrightarrow p \vee (\forall x)q(x)$.
10. $(\forall x)(p \wedge q(x)) \Leftrightarrow p \wedge (\forall x)q(x)$.
11. $(\exists x)(p \vee q(x)) \Leftrightarrow p \vee (\exists x)q(x)$.
12. $(\exists x)(p \wedge q(x)) \Leftrightarrow p \wedge (\exists x)q(x)$.
13. $(\forall x)p(x) \Leftrightarrow \neg(\exists x)\neg p(x)$.
14. $(\exists x)p(x) \Leftrightarrow \neg(\forall x)\neg p(x)$.
15. $\neg(\forall x)p(x) \Leftrightarrow (\exists x)\neg p(x)$.
16. $\neg(\exists x)p(x) \Leftrightarrow (\forall x)\neg p(x)$.
17. $(\forall x)(p \longrightarrow q(x)) \Leftrightarrow p \longrightarrow (\forall x)q(x)$,

    in monadic logic corresponding to in monadic algebra,

    $$(\forall x)(p \longrightarrow q(x)) = (\forall x)(p' \vee q(x))$$
    $$= p' \vee (\forall x)q(x) = p \longrightarrow (\forall x)q(x).$$
18. $(\exists x)(p \longrightarrow q(x)) \Leftrightarrow p \longrightarrow (\exists x)q(x)$.

    " by similar method in 17".
19. $(\exists x)(p(x) \longrightarrow q) \Leftrightarrow (\forall x)p(x) \longrightarrow q$,

    in monadic logic is counterpart of monadic algebra,

    $$\begin{aligned}(\exists x)(p(x) \longrightarrow q) &= (\exists x)((p(x))' \vee q) \\ &= (\exists x)(p(x))' \vee q \\ &= \sup_{x\in X}\{p'(x)\} \vee q \\ &= \left(\inf_{x\in X}\{p(x)\}\right)' \vee q \\ &= ((\forall x)p(x))' \vee q \\ &= (\forall x)p(x) \longrightarrow q.\end{aligned}$$
20. $(\forall x)(p(x) \longrightarrow q) \Leftrightarrow (\exists x)p(x) \longrightarrow q$,

    " by similar method in 19".

VI. SOME LOGICAL CONCEPTS SUCH AS DEDICATION

Let B be a Boolean Algebra. Suppose that the elements of B represent propositions (or statements) of logic. The set of all provable (satisfiability or true) propositions is a filter of B,because conditions 1 and 2 of the definition of the filter are sissified [10]. A proposition p is refutable (unsatisfiable or false) if p' is provable. The set of all refutable propositions is an ideal of B.since the definition of ideal is satisfied [10]. Dedication in propositional logic can be performed algebraically if we consider the premises of an argument form are elements of filter F and deduced that the conclusion belongs to the filter F.Also all inference rule of propositions logic can be done algebraically.

Example (1). Consider $p \longrightarrow q,\ q \vdash q$. In corresponding Boolean algebra B. Let F be a filter such that $p \longrightarrow q$, $q \vdash q \in F$, therefore $(p' \vee q) \wedge p \in F$. Hence

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$(p' \vee q) \wedge p = p \wedge q, but\ p \wedge q \leq q. so\ q \in F.$ Therefore if $p \longrightarrow q$ and p are provable, then so is q. Let M be a monadic algebra i.e. M is a Boolean algebra with quantifier $\exists$. A subset I of M is a monad ideal if I is a Boolean ideal and $\exists(I) \subseteq I$.A subset F of M is a monadic filter if F is a Boolean filter and $\forall(F) \subseteq F$.

VII. CONCLUSION

In this paper, we studied the algebraic method of propositional and monadic logic, and the treatment of logic by method of algebraic. In future work, we will study polyadic logic and algebraic methods.

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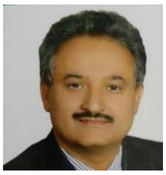

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