

# A Model for Combining Allegorical Mental Imagery with Intuitive Thinking in Understanding the Limit of a Function



Ahmad Hedayatpanah Shaldehi, Marziyeh Hedayatpanah Shaldehi, Badriyeh Hedayatpanah

**Abstract:** *The main purpose of this article is to use a combination of mental imagery and puppet allegory to intuition the concept of limit for learners. Usually to start teaching, teaching the limit of functions, there are several methods, but most instructors use the Epsilon and Delta formula. Obviously, starting with teaching with formulas, it is not easy to explain the concept of limit. Because, learners achieve a deep understanding of the limit, when the concepts become intuitive for them. Intuition of the limit of functions, with proper mental imagery and allegory, can make understanding the limit easy and enjoyable. Obviously, following the optimal understanding of the limit, understanding, Neighborhood. Derivation and integrals, for learners It becomes easier. The use of a combination of mental imagery with puppet allegorical patterns to intuitive or sensory thinking that is the result of three decades of experience and teaching of the author, to be available to learners and educators. In this way, very soon, learners (especially non-mathematics learners), especially in high school classes, or the first years of university, they manage to have a deep understanding of the limit, Because one of the ways to teach mathematics is to use the properties of mental imagery to be objective or sensory through allegory, which has a high effectiveness and feedback in understanding the limit. In this article functions limit education is considered by combining mental and sensory imagery through allegory or simile with six puppet images. The experiences in this research can be used by teachers of teacher training centers, teachers and high schools and universities and designers of educational materials.*

**Keywords;** *Limit, Allegory, Teaching, Mental Imagery, Intuitive (Sensory).*

## I. INTRODUCTION

Mathematics is a science that generally deals with abstract or subjective ideas. Math information is displayed, with numbers, words and other symbols. These mathematical symbols are often represented visually as the formation and development of a model or its placement with other models.

It is important to be able to visualize mathematical concepts better. Visual thinking means at least three things:

- A) Perception means awareness through the senses: (sight, hearing, taste and smell) as well as through movement and change of body position.
- B) Simulation means shaping a sensory perception in the mind or body.
- C) Image means showing understanding through design, diagram, model or revision.

When visual thinking grows, it can play an important role in developing mathematical understanding and creatively application of mathematics in other areas. Visual display is a powerful way for Student access to mathematical mental ideas. (Etesami, 2019[1]). In fact, one of the goals of math education, is to develop the skills that students need to solve real-life problems. (Scarlek Keith, 2001[2])." Skills in mathematics namely the ability to solve problems, find proofs, critically analyze, conclusions, use the simplest possible methods of mathematical structures, and finally recognize mathematical concepts and specific situations and cases" (Polya George ,2013[13]). and new innovations and schemas. To succeed in this work, it takes a long time for the teacher to gain the necessary experience for successful teaching, before referring to the early years of his service, to realize his tangible progress in various fields of education and teaching. This improvement is achieved when the teacher can use methods and techniques to provide acceptable content and learners' satisfaction. (Boris Joyce Marshall, 1996[4]).

In presenting new and innovative methods, the teacher must have the appropriate mechanism to act on the basis of accurate and comprehensive knowledge. "Creating new methods not only helps to teach math, but also creates a challenging state and also creates a passion for teaching, so if it does not materialize, it may seem tedious to the teacher." [Otto C Bassler and Kolb (1971). [5)]. When the teacher has sufficient knowledge and experience, he can have a successful teaching method, considering the factors mentioned above, he seeks these successes, which result in educational innovations, because "the reality is that the use of the new method requires practice and practice. Repetition (Boris Joyce Marshall, 1996[4]). As mathematics textbooks change from elementary to advanced levels, so do technical subjects from intuitive to abstract. Or sensory and, finally, turn to verbal recollection. Obviously, teaching and understanding more technical mathematical topics requires a variety of ideas and skills, in particular, to understand abstract concepts.

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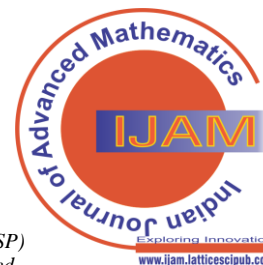
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Due to the necessity of the subjects at higher levels, teachers sometimes resort to reasoning as necessary.

## II. METHODOLOGY

The research method is library. It is complemented by more than 30 years of experience in teaching, writing and researching the author.

### 2.1. Argument in mathematics

Proponents of the reasoning method believe that mathematics is based on logic and its purpose is to strengthen the power of reasoning (Shokouhi, 1985[6]), which is usually presented in three methods: deductive, inductive and allegorical.

#### 2.1.1. Deduction (syllogism)

Sometimes the mind, from general theorems, reaches partial conclusions, Or the mind came from general theorems to less general theorems, but Analogy in math is that, from the principles laid down at the beginning of this science, Theorems whose results are essential to those principles. Be pulled out. In other words; the method of constructing a science is called the deductive method by the principles of the subject. (polya George,1964 [7]).

#### 2.1.2. Induction

It is an argument that the mind, relying on experience, reaches from a less general theorem to a more general theorem, Induction is complete and incomplete. In mathematics, imperfect induction is commonly used. (polya, 1964[7]).

It is an argument that the mind, relying on experience, reaches from a less general theorem to a more general theorem, Induction is complete and incomplete.

#### 2.1.3. Allegory

An allegory is a statement that says that two objects, process, or two events are similar. Researchers use them to come up with ideas for facilitating logical comparisons. Allegories convey information about existing patterns in data by referring to something known, or an experience familiar to the reader. (Newman, 2016 p. 512[8]). Allegories are the relative similarities between phenomena, and relatively different situations, which make possible more inferences, and allegory is the reference to a kind of similarity, in which the same relational structure exists in two different situations. Jenter, 1983 & Jenter and Landers 1985: tested a wide range of stories and found that superficial similarity was the best predictor of memory achievement. Similarity in relational structure, on the other hand, is the best predictor of inference accuracy classifications. They are also superior in not being deceived by superficial similarities, and the resulting people. Given the importance of allegory, in transmission, it can be said that an important topic of research, exploration and discovery of determining factors is the achievement of allegories (Eysenck, 2000[9]). Psychologists study allegories for several reasons: first, allegories are important in learning. Second, allegories are important in the problem-solving process (Holly Oak, 1985[10]). The third reason for studying allegory and similarity is that they seem to be the foundation of other cognitive processes. Most patterns are based on allegory; the role of mathematical allegories in scientific modeling, which indicates the direct impact of these

allegories on the model of their phenomena. (Ebadi & bakhshipour, 2020[11]). Some cognitive scientists believe that allegory reasoning is the basis for the development of critical thinking skills. (Hunt, 1982) Probably, there is no better way than to use a correct allegory to give life to an abstract concept (Critical thinking) and there is no accessibility for students Allegory is a conceptual structure, which purposefully plays the role of Transfer of concepts; in fact, it is a movement from darkness to light Allegory is a fundamental structure, which conceptually brings meaning to the human mind and ultimately turns it into action (Eysenck& Smith, 2000[9]).

It can be said that allegories reconstruct the mental and social conditions of children (Fisher, 2007). Bourdain showed that, in order to improve the educational system in schools, critical thinking skills must be taught. And came to the conclusion that; The best way to teach critical thinking is to teach allegory. (Yar Mohammadi, Farhai and Yaghoubi, 2017[12]). Also, George Polya (2013) has his ideas about allegory: "Allegory is a kind of similarity, things that are similar in some ways agree with each other, similar things agree in some of the relations of the corresponding parts" (Polya George 2013[3])." Teachers need to identify and design Allegories that which uses the previous knowledge of students. Allegory helps students use their prior knowledge. However, since allegories are not the subject of the lesson, teachers need to make sure that the allegories The allegories they choose are appropriate and point out the limitations of the allegories carefully. (Glover et al. 1977[13]). Sometimes allegory alone does not meet the needs of the audience, especially in technical mathematics topics at higher levels, which require illustration. Thus, allegorical mental images, if combined with intuitive, sensory, or revelatory thinking, will yield desirable, and more effective, results.

### 2.2. Sensory method or intuitive thinking

Proponents of the intuitive method are influenced by philosophers who see emotion as the source of all human knowledge. In the opinion of these scientists, The human mind, In the beginning, it is like a whiteboard That nothing is written on it, but Gradually, experience maps out information in it.

Experience is of two kinds, one emotional, which comes from foreign objects, the other internal observations, which gives thought and reason to her him, and reason nurtures the information in the mind, and it is one of the sentence of Janlak, that "nothing is in the intellect except what has been sensed." This method is also known as the sensory method (intuition) (Shokouhi,1985[6]). Combining mental imagery, and allegory and transmitting it to the senses, is very effective in teaching mental and complex concepts, especially mathematics.

### 2.3. Mental imagery

Mental illustration is a psychic skill that, by consciously using the imagination, creates and reconstructs explicit mental images in the brain.

Individuals can mentally create images of simple subjects, such as shapes, or mental images of images to recall complex life-related events. (Haseli, 2019[14]).

One of these methods is to see images. Humans, by looking at pictures, drawings, and images, transfer them to the mind and archive them as images in their own brains. Thus, images play an important role in learning, activating the faculty. Learn in the mind. One way to solve problems and learn is through learning style. In the visual learning style, 65% of people and students are more interested in learning when they see a picture. ; (Nazari, 2016[15]). Using mental imagery in mathematics education can provide a way to better learn this lesson. And made it easy to succeed in mathematics, and to solve problems; and, in fact MENTAL imagery is an appropriate method of representing mathematics. The use of mental imagery in teaching and learning mathematics can have a profound effect on understanding, learning, and problem-solving skills. (Isa Khani & Shamsi, 1400[16]). One of these methods is to see the pictures. Humans, by looking at pictures. And images, transmit them to the mind, and archive them in their own brain in the form of images. So images play an important role in learning, and they activate the learning power in the mind. One set of ways to solve a problem, and to learn, is the style of learning. In the visual learning style, 65% of people, and students, learn with more interest when they see a picture. ; (Nazari, 2017[15]). Strategies, and programs. Students who use visual theories accurately are six times more likely than students who do not use them to solve math problems correctly. Students who use visual theaters to solve problems verbally are more likely to have problems to solve accurately. (Etesami, 2019[1]). Specialists, given the need for models, and images, on which mathematical thinking can be mounted. Robert Summer, a specialist in psychology and environmental studies at the University of California, said: When a mathematical phrase is uttered without any image or connection, it despondent the apparently, these words are expressed in a foreign language. In fact, mathematics is often taught as a foreign language only through arbitrary connections between symbols and objects. The problem is not just the symbols themselves, but that our mathematics education separates numbers from the realities of life and makes mathematics intangible. (The same source). Considering what has been said, the main purpose of this article is Teach basic topics, limit functions, relying on experience, using a combined model of mental imagery, and an allegorical model, through six puppet images, and transferring it to objectivity (intuitive thinking). , Which creates a pleasant environment for learners, very soon, they understand the concept of limit.

### III. MINOR OBJECTIVES

- An allegorical image to teach a variety of approaches.
- Allegorical image to teach images orientation.
- Allegorical mental imagery of the basic topics of limited. Education, intuitively through images.
- The relationship between allegorical and intuitive. Mental imagery as a function.
- Transfer of experiences.

### IV. THE CONCEPT TO TEND (TO APPROACHES).

#### 4.1. To tend from the right

Answer to Question (1) Consider a piece of paper the length of a unit, Figure (1, a), divide it into two equal parts, set aside half of it (Figure 1, b) and consider the remaining half to continue the discussion. Get.

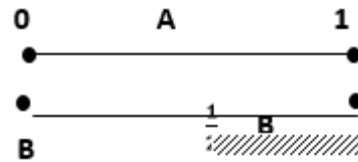


Figure 1.

. Divide the rest ( $\frac{1}{2}$ ) into two equal parts and leave half of this part aside. And the other half (Keep the new remnants to continue the discussion) and continue this process, It can be seen that the remains are getting smaller and smaller. Figure (2) shows the work process.

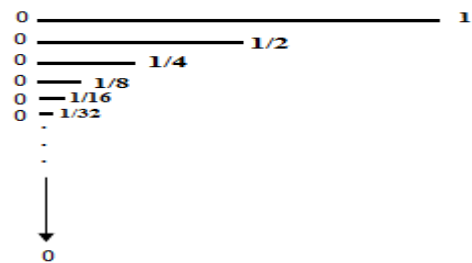


Figure 2.

Finally, the question may happen: Is the piece of paper finished with consecutive halving?

Answer: No, because it seems that the paper is physically it is coming to an end. But if you have the tools and the ability, you can make it smaller and smaller. Confirmation of this claim is evident in the following the following math sequence .

$$1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \frac{1}{64}, \frac{1}{128}, \frac{1}{256}, \frac{1}{512}, \dots \rightarrow 0$$

As it follows from the sequence above, the sequence gets closer and closer to zero, but never becomes zero, in mathematical terms this is called tend, and so it is written  $x \rightarrow 0$  and read: x tends to be zero.

Note: In the example above, the number zero is not absolute.

#### 4.2. To tend from the Left

Question 2) Are the direction of the arrow (to tend from the right ( $\leftarrow$ )) different from the direction of the arrow to the left ( $\rightarrow$ )? Answer: Yes, For example: x may be closer to a number to tend 2 from the right, i.e. as follows (Figure 3; A)



Figure 3.



And this means that if  $x$  tends to the number 2 from the right, purpose, numbers larger than 2 that tends to 2, that is, a particle greater than 2 absolute, and denotes

$x \rightarrow (2^+)$ . Give

If  $x$  tends to the number 2 from the left (Figure 3; B), purpose, numbers smaller than 2 that tends to 2. That is, a particle is smaller than 2 and is denoted by  $-x \rightarrow 2^-$ .

4.3. Intuitive allegorical mental imagery for the concept of tends to (Answer to Question (2))

Intuitive imagery with the following allegory helps to understand more of the above

A) In the mental balance image .The Strong number two ( $2^+$ ) is heavier or greater than the absolute number two, i.e. (2). Figure (4a)

B) In the image of the mental balance, absolute (2) is heavier or greater than the weak number two ( $2^-$ ). Figure (4b).

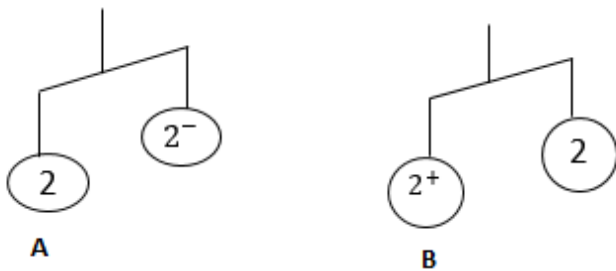


Figure 4.

So in the result (the same analogy about the  $2^+ > 2 > 2^-$ ) number zero. ( $0^+ > 0 > 0^-$ )

4.4. Mental imagery before teaching (answer to question (3))

| The position of the two hands | Profile of dolls         |           |            | Puppet character |
|-------------------------------|--------------------------|-----------|------------|------------------|
|                               | Size of two hands        | left hand | Right hand |                  |
| complete                      | equal                    | have      | have       |                  |
| incomplete                    | The right hand is bigger | have      | have       |                  |
| incomplete                    | Left hand is bigger      | have      | have       |                  |
| incomplete                    | Unequal                  | —         | لارد       |                  |
| incomplete                    | Unequal                  | have      | —          |                  |
| -                             | -                        | -         | -          |                  |
| zero                          | equal                    | have      |            |                  |

V. THE CONCEPT OF LIMIT WITH INTUITIVE IMAGERY

One of the most important topics in general mathematics is the limit. Because sometimes, it is necessary to study functional behavior near the point of study and observe what changes in  $y$  when  $x$  (or the independent variable) approaches

a point. Or a dependent variable), if a change occurs, how? What will be the answer? The concept of limit. Is now explained by example. Just as we are dealing with types of functions, we are also dealing with types of limit. Limits of first, second, and third degree functions or more, polynomial functions, limit of fractional functions, limit of finite derivation, limit of trigonometric functions, limit of exponential functions of logarithmic, limit of radical functions, limit of integer functions, absolute value and polynomial types. But with all the descriptions in this article, allegorical illustration has been done intuitively.

5.1. Limit of a function

The concept of subordination; Example (1); Consider the function  $f(x) = 2x - 1$ , assuming that  $x$  takes numbers very close to the number 2 from the left, as in Table (1)

Table (1)

|        |                 |                   |                    |                     |                 |
|--------|-----------------|-------------------|--------------------|---------------------|-----------------|
| $x$    | $\rightarrow 1$ | $\rightarrow 1.9$ | $\rightarrow 1.99$ | $\rightarrow 1.999$ | $\rightarrow 2$ |
| $f(x)$ | $\rightarrow 1$ | $\rightarrow 2.8$ | $\rightarrow 2.98$ | $\rightarrow 2.998$ | $\rightarrow 3$ |

Table (1)

Table (1) shows that,  $x$ , never reaches the number 2, but gets closer and closer to it (the number 2) from the left. Here it is said: If  $x$  from the left, if desired, tends to the number 2, its image, i.e.  $f(x) = 2x - 1$ , is close enough to the number 3. The above sentence is written with a mathematical symbol in the following form

$$\lim_{x \rightarrow 2^-} 2x - 1 = 3$$

And is read, the left limit is a function of the number 3, if  $x$  tends to the number 2 from the left.

In the same way we find that

$$\lim_{x \rightarrow 2^+} 2x - 1 = 3$$

5.1.1. Limit on the right of a function

A) Right limit) Whenever  $x$  tends to it ( $\alpha$ ), for values greater than  $\epsilon$ , the value of the function,  $f(x)$ , approaches a certain value such as  $L1$ , which gives  $L1$  the right limit of the function at point.

Example (2):

$$\lim_{x \rightarrow 3^+} 2x + 5 = 11 \quad : \quad \text{Right limit}$$

5.1.2. Limit on the left of a function

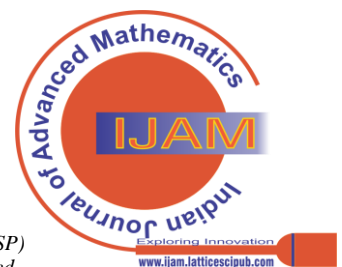
B) Left limit) Whenever  $x$  tends to ( $\alpha$ ), for values less than  $\epsilon$ , the value of the function,  $f(x)$ , approaches a certain value such as  $L2$ , which is called the left limit of the function at the point.

$$\lim_{x \rightarrow 3^-} 2x + 5 = 11$$

Note: When the left and right limits are equal in a function, we say that the function has a limit. Like the example (2) above.

5.1.3. Limit of the function

(Limit of the function) Suppose that the function  $f$  in the neighborhood  $\alpha$ , probably except in itself ( $\alpha$ ) (defined),



the value of L is called the limit of the function at point  $\alpha$ , when x arbitrarily approaches, the corresponding values of f(x) is close enough to the number L. Approach, they write.

$$\lim_{x \rightarrow \alpha} f(x) = L$$

**Example (3):** limit  $f(x) = 2x + 5$  when  $x \rightarrow 3$  : ( It has limit )

**Example (4):**

$$\Rightarrow h(x) = \begin{cases} 4 - x^2 & x \leq 1 \\ 2 + x^2 & x > 1 \end{cases}$$

$$\{L_1 = L_2 = 3\}$$

$$\lim_{x \rightarrow 1^-} h(x) = \lim_{x \rightarrow 1^-} 4 - x^2 = 3$$

$$\lim_{x \rightarrow 1^+} h(x) = \lim_{x \rightarrow 1^+} 2 + x^2 = 3$$

**Example (5)**

$$f(x) = \begin{cases} -1 & x < 0 \\ 0 & x = 0 \\ 1 & x > 0 \end{cases} \Rightarrow$$

$$\{L_1 \neq L_2\}$$

$$\lim_{x \rightarrow 0^-} f(x) = -1$$

$$\lim_{x \rightarrow 0^+} f(x) = +1$$

**Example (6)**

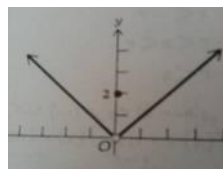
$$\lim_{x \rightarrow 4} f(x) = \begin{cases} x - 3 & x \neq 4 \\ 5 & x = 4 \end{cases}$$

$$\lim_{x \rightarrow 4} f(x) = 1$$

**Example (7)**

$$f(x) = \begin{cases} |x| & x \neq 0 \\ 2 & x = 0 \end{cases}$$

$$\lim_{x \rightarrow 0^+} g(x) = \lim_{x \rightarrow 0^+} x = 0$$

$$\lim_{x \rightarrow 0^-} g(x) = \lim_{x \rightarrow 0^-} -x = 0$$


**Note:** Epsilon and Delta can now be used easily

### 5.1.4. Defining the limit through Epsilon ( $\epsilon$ ) and Delta ( $\delta$ )

Suppose f is a function defined at all points in an open interval, such as I, which contains the number  $\alpha$  (except possibly  $\alpha$  itself), then the limit f(x) when x tends to  $\alpha$ , is equal to L.

In other words, if  $\lim_{x \rightarrow \alpha} f(x) = L$

$$(\forall \epsilon > 0, \exists \delta > 0 : 0 < |x - \alpha| < \delta \Rightarrow |f(x) - L| < \epsilon)$$

We read that for every positive epsilon, any small size, there is a positive number such as delta that the absolute value of x minus alpha ( $\alpha$ ) is smaller than the delta, then the absolute value of f(x) minus L is less than epsilon ( $\epsilon$ ).

Attention: ( $0 < |x - \alpha| < \delta$ ) . This means that x tends to alpha but will never become itself ( $\alpha$ ), so their difference, no matter how very, very small, makes sense.

Attention: The following three terms are equivalent:

$$\lim_{x \rightarrow \alpha} |f(x) - L| = 0 \quad \& \quad \lim_{x \rightarrow \alpha} (f(x) - L) = 0$$

$$\& \quad \lim_{x \rightarrow \alpha} f(x) = L$$

**Example (9):** Prove that using the limit definition

$$\lim_{x \rightarrow 3} 4x - 1 = 11$$

$$\forall \epsilon > 0, \exists \delta > 0 : 0 < |x - \alpha| < \delta \Rightarrow |f(x) - L| < \epsilon$$

$$\forall \epsilon > 0, \exists \delta > 0 : 0 < |x - 3| < \delta \Rightarrow |4x - 1 - 11| < \epsilon$$

$$\forall \epsilon > 0, \exists \delta > 0 : 0 < |x - 3| < \delta \Rightarrow |4x - 12| = 4|x - 3| < \epsilon$$

$$\forall \epsilon > 0, \exists \delta > 0 : 0 < |x - 3| < \delta \Rightarrow |x - 3| < \frac{1}{4} \epsilon$$

$$\Rightarrow \delta = \frac{\epsilon}{4}$$

With the condition ( $\delta$ ed that) it is proved  $\frac{\epsilon}{4}$

$$\lim_{x \rightarrow 3} 4x - 1 = 11$$

Or

$$\delta = \epsilon$$

**Example (10):** How much to choose the amount of delta ( $\delta$ ) to have

$$\lim_{x \rightarrow 1} 5x - 3 = 2$$

$$\forall \epsilon > 0 \exists \delta > 0 : 0 < |x - \alpha| < \delta \Rightarrow |f(x) - L| < \epsilon$$

$$: 0 < |x - 1| < \delta \Rightarrow |5x - 3 - (2)| < \epsilon$$

$$: 0 < |x - 1| < \delta \Rightarrow |5x - 5| < \epsilon$$

$$: 0 < |x - 1| < \delta \Rightarrow 5|x - 1| < \epsilon$$

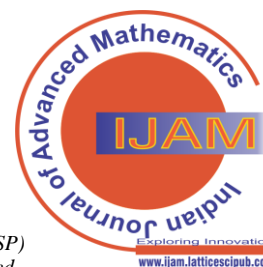
$$: 0 < |x - 1| < \delta \Rightarrow |x - 1| < \frac{\epsilon}{5}$$

$$\Rightarrow \delta = \frac{\epsilon}{5}$$

The following books have been used to express the concepts of had: Leithold, (2009 [17]) & Silverman, (2012 [18]) & Thomas, (2004 [19])

## VI. RESULT

The summary of the above results can be seen in the form below. According to what was taught about the limit of functions



| Image | The corresponding mental image |                             |                   |                    | Corresponding examples   | Puppet character |
|-------|--------------------------------|-----------------------------|-------------------|--------------------|--|------------------|
|       | result                         | Limit size                  | limit on the left | limit on the right |  |                  |
|       | There is a limit               | equal                       | 3                 | 3                  | $h(x) = \begin{cases} 4 - x^2 & x \leq 1 \\ 2 + x^2 & x > 1 \end{cases}$ |                  |
|       | There is no limit              | The right limit is larger   | -1                | 1                  | $f(x) = \begin{cases} -1 & x < 0 \\ 0 & x = 0 \\ 1 & x > 0 \end{cases}$  |                  |
|       | There is no limit              | The left limit is larger    | 4                 | 2                  | $h(x) = \begin{cases} x + 3 & , x \leq 1 \\ 3 - x & , x > 1 \end{cases}$ |                  |
|       | There is no limit              | It has only the right limit | —                 | 0                  | $\lim_{x \rightarrow 0} \sqrt{x}$  |                  |
|       | There is no limit              | It has only the left limit  | 0                 | —                  | $\lim_{x \rightarrow 1} \sqrt{1-x}$                                      |                  |
|       | There is no limit              | —                           | —                 | —                  | $\lim_{x \rightarrow 4} \sqrt{-(x^2)}$                                   |                  |

### VII. CONCLUSION

It was said that mathematics is a science that; Solves problems of abstract ideas. Information in mathematics can be represented by numbers, words and other symbols; And this information is often presented visually as the formation and development of a model or its placement with other models. It is important to be able to visualize mathematical concepts better

2 - In this article, first to the mental and allegorical puppet illustration of the limit of functions, then with the help of taking the shapes of functions, the limit can be easily taught to learners.

3- Visual thinking means understanding, simulating and illustrating Rad himself.

4- The findings of cognitive sciences have shown that, if, patterns are presented with reference to allegories that are more familiar to the mind. It is very effective in explaining scientific concepts and formulating theories by sensory or revelatory methods.

5- Therefore, what was stated in this article and the result was obtained, the teaching of limit with the pattern of puppet mental imagery, and the allegorical method is easily possible and applicable. The result is sustainable learning, especially in the areas of knowledge, understanding and analysis for learners. It has a wonderful.

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### AUTHORS PROFILE



#### Ahmad HyatPanah Shaldehi

- Place of birth: 1957, Iran, Gilman
- Field of study: Mathematics (diploma to doctorate)
- Skills and interest: Statistics and research methods in the behavioral sciences of chord and
- Teaching: 45 years (university and education)

#### Researches:

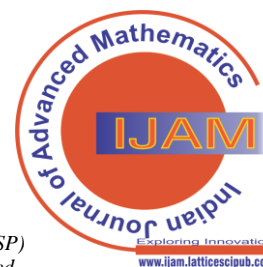
- Conductor of various researches: 45 cases

Published books: 16 items, like:

- Cognitive Style (FI&FD) Mathematical According Bloom's Taxonomy, Publisher (lap) in Germany
- General Mathematics(1):ISBN : 978-964-9592-82-4 Publisher : Daryaye Danesh
- General Mathematics(2):ISBN : 978-964-6977-39-1 Publisher : Abrang
- How we research? ISBN :978-964-9592-85-5 Publisher: Daryaye danesh
- Statistics and Probability ISBN: 978-964-95928-3-1 Publisher : Daryaye danesh
- Pre Mathematics . ISBN : 978-964-9592-84-8 Publisher : Daryaye danesh
- Why learn math? ISBN: 964-6677-12-x : Publisher : Abrang danesh
- The importance of prayer from a research perspective, ISBN: 978-600-7223-11-6 Publisher : Daryaye danesh

Awards, 56items:

1. The Best researcher Iranian 2012
  2. The best University2012 of superior technical and professional Researchers
  - 3) The Best University2013 of superior technical and professional Researchers
  - (4 The Best researcher Iranian (Guilan) 2012
  - (5TheBest researcher Iranian (Guilan) 2013
  - 6) The Sample Teacher Iranian 2003
  - 7) The Sample Teacher Iranian 2006
  - 8) The Sample Teacher Iranian 2012
  - 9) Author of the article Top in Iran2000
  - 10) Author of the article Top in Iran2004
  - 11) Author of the book Top Iran 2019
  - 12) Author of the book Top Iran 2018
  - 13) Author of the book Top Iran ,...
- Workshop: 53 items



-Judging articles: Over 1300 cases (provincial, national and international)  
Articles: like  
Presenting papers in national and international conferences: 51 cases  
-- Published articles: 102 cases in Iran and 24 cases outside Iran  
1. Fuzzy approach to Likert Spectrum in Classified levels in Surveing research( Scopus&Esci) citation(21)  
-The Journal of Mathematics and Computer Science 2 (2), 394-401 ,2011.  
2. Study and investigation of the problems and learning disorders of students by various cognitive styles in mathematics course at Rasht shahid chamran higher education center.  
The Journal of Mathematics and Computer Science 1 (3), 216-229 2010.  
3. Using Eta ( $\eta$ ) correlation ratio in analyzing strongly nonlinear relationship between two Variables in Practical researches ) Scopus&Esci) citation(9 )  
AH Shaldehi  
Journal of mathematics and computer science(tjmcs) 7 (3), 213-220 2013.  
4. Students' Field-dependency and Their Mathematical Performance based on Bloom's Cognitive Levels  
R farhad, Alamolhodaei Hassan , hedayatpanah Ahmad  
Korean Society of Mathematical Education 15 (4), 373-386 2011.  
- 5. Infinite teaching ( $\mathbb{R}$ ) in collection and ( $\mathbb{R}^*$ ) by allegory and conformity  
Indian Journal of Advanced Mathematics (IJAM) Volume-1 Issue-1, April 2021.  
Ahmad hedayatpanah shaldehi / Marziyeh hedayatpanah shaldehi / Kolachahi Sabet Mohammad Taghi/ Mohammad Saeed hedayatpanah shaldehi  
6- Comparative Analysis of Similarities and Differences between Null Hypotheses, Assumption of Breach;  
Indian Journal of Advanced Mathematics (IJAM) ISSN: 2582-8932, Volume-1 Issue-2, October 2021: pp:20-26  
Ahmad Hedayatpanah Shaldehi, Mohammad Saeed Hedayatpanah Shaldehi, Marziyeh Hedayatpanah Shaldehi  
7- Effectiveness of Oral Tests in Improving Learners' Mathematical Performance  
Ahmad Hedayatpanah Shaldehi  
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8....



**Marzieh Hedayat Panah Shaldehi**

Place of birth: Iran, Gilan, Rasht  
Bachelor: Computer Science  
Master: Information Technology Engineering  
Research: 4 cases  
Articles: 22 items. liik

The Survey of student's mathematics Calculations disorder" in International Journal of Advance Research in Engineering and Applied Sciences(jjareas) , Volume2, Issue1,Pages 89-96, 2015  
2. "A Study on the Characteristics of a Virtual Classroom" in International Journal of Innovation and Research in Educational Sciences, Volume 1, Issue 2, ISSN (Online): 2349-5219, 2014  
3. "Performance Comparison of the mathematical skills of Computer and Electronics students in the Chamran college technical rasht" in Journal of Progressive Research in Mathematics (JPRM), An International Journal, Volume 1, Issue 1, December, 2014  
Teaching: 7 years in different universities (technical and professional, free, Payam Noor, Simaye Danesh and Koushiar).  
Job: University lecturer  
Book printing: 2 items, like  
- Design and implementation of electronic library. ISBN: 978-600-7223-04-8 Publisher: Daryaye danesh Publisher: Daryaye Danesh  
- Instructor of the workshop 6 case  
-Awards: 4 items



**Badriyeh Hedayat Panah Shaldehi**

Place of birth: Iran, Gilan, Rasht  
Bachelor: mathematics  
Master: Master of Quranic Sciences.  
Research: 4 cases:  
Articles: 13 items.  
overview on teaching methods of mathematics.

ahmad hedayatpanah shaldehi / hedayatpanah Badriyeh.  
Ijetsi 1 (01), 48-58 pub med  
Job: educationalist .  
Book printing: 4 items,  
Instructor of the workshop: 6 case  
Awards: 10 items

